GOVERNMENT OF TAMILNADU
DIRECTORATE OF TECHNICAL EDUCATION
CHENNAI – 600 025
STATE PROJECT COORDINATION UNIT

Diploma in Instrumentation and Control Engineering

Course Code: 1042
M – Scheme
e-TEXTBOOK
on

ELECTRICAL CIRCUITS AND MACHINES

for
III Semester DICE

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M-SCHEME

(Implements from the Academic year 2015-2016 onwards)

Course Name :**DIPLOMA IN INSTRUMENTATION AND CONTROL ENGINEERING**
Course Code : **1042**
Semester : **III**
Subject Code : **34232**
Subject : **ELECTRICAL CIRCUITS AND MACHINES**

Teaching and Scheme of Examinations: No. of Weeks per Semester: **15**

Subject Code & Name	**Instruction**		**Examination**			
	Hours/ Week	Hours/ Semester	Marks			Duration in Hours
Electrical Circuits and Machines	6	90	Continuous Assessment	End Semester Examination	Total	3
			25	75	100	

Topics and Allocation of Hours:

Unit No	**Topics**	**Hours**
I	DC circuits and DC Networks Theorems	16
II	AC circuits	16
III	Resonance and Three phase AC circuits	16
IV	DC Machines and AC Machines	16
V	Transformers	16
	Test & Revision	10
	Total	90

RATIONALE:

The fundamental knowledge about Electrical circuits both AC and DC is essential for diploma holders. The working principles of DC machines, AC machines and transformers are prerequisite for technicians in their workplace. This subject helps in this way.

OBJECTIVES:

- Define voltage, current, resistance, resistivity, power, energy and their units.
- State and explain ohm's law and Kirchoff's law and solve simple problems
- Derive equivalent resistance of series and parallel circuits
- Solve problems in mesh current and nodal voltage method
- State and explain super position theorem, Thevenin's theorem, Norton's theorem
- and maximum power transfer theorem and solve problems in theorems
- Explain 3ϕ power measurement by two watt meter method
- Explain constructional details of dc machines
- Explain the construction, working and starting methods of 1 ϕ &3 ϕ induction motors
- Explain the principle and working of different types of induction motor
- Explain the principle and working of transformer

ELECTRICAL CIRCUITS AND MACHINES

DETAILED SYLLABUS

Unit	Name of the Topic	Hours
I	**DC Circuits and DC Network Theorems** Concept of electrical quantities – Voltage – current – resistance – power – energy – ohm's law – Kirchhoff's laws- Resistances in series –Resistances in parallel – series parallel circuits- Thevenin's, Norton's, Super position and maximum power transfer theorems – Statement and explanations – Simple problems.	**16**
II	**AC Circuits** AC fundamentals – AC waveform – sinusoidal and non-sinusoidal – period – frequency – cycle – amplitude – phase – peak value – average value – RMS value (effective value) – form factor – crest factor AC Through pure resistor, inductor and Capacitor – Concept of impedance – RL, RC and RLC series circuits vector diagram. – derivation – simple problems. Power in AC circuits – power factor–power triangle- simple problems. Introduction of Harmonics -Effects of Harmonics	**16**
III	**Resonance and 3 ϕ AC circuits** Resonance – condition for resonance – series and parallel resonance – resonance curve – effect of resistance on resonance curve – selectivity – Q factor and bandwidth – applications of resonance – simple problems in resonance. Concept of 3 ϕ supply – line and phase voltage and current in star and delta connected circuits – three phase power – Measurement of three phase power by two watt meter method – simple problems – advantages of three phase over single phase system.	**16**
IV	**D.C Machines and A.C Machines** DC machines – Types – constructional details of DC machines – DC generators – principle – types – emf equation – characteristics of shunt, series and compound generators DC motor – types – motor action – back emf – torque speed characteristics – starting of motors using 3 and 4 point starters – speed control of DC motor-applications. AC machines – 3 ϕ alternator – construction and working – relation between speed and frequency. 3 ϕ Induction motor – construction – types – principle of operation – methods of starting of 3 ϕ induction motor – slip. Single phase induction motor – principle of operation – capacitor start - motors – Applications – principle of operation -Stepper motor .	**16**
V	**Transformers** Transformer – Ideal transformer – principle of working –constructional details – emf equation – turns ratio – core loss and copper loss – efficiency – regulation – OC and SC tests problems. Transformer on No load – Transformer on load – condition of maximum efficiency – All-day efficiency(simple problems). Auto transformer – construction and working – applications.	**16**
	Revision and Test	**10**

Text books:

1. Theraja. B.L., “A text book of Electrical Technology, Vol. I & II”, S.Chand & Co., Year
2. Nagoor Gani, “Circuit Theory”, RBA Publications.

Reference books:

1. Arumugam & Prem kumar, “Circuit Theory”, Khanna Publishers, Year
2. Louis M.M., “Elements of Electrical Engineering”, Khanna Publishers, Year
3. Gupta M.L. ,S.K.Kataria & Sons, “Elementary of Electrical Engineering”, Year

ELECTRICAL CIRCUITS AND MACHINES

UNIT – 1 - DC CIRCUITS AND DC NETWORK THEOREMS

1.1 CONCEPT OF ELECTRICAL QUANTITIES

Electrical Energy is measured by quantities like electric charge, current and potential difference (voltage).

Charge :

Electric charge is the physical property of matter that causes it to experience a force when close to the other electrically charged matter. There are two types of charges Positive and Negative. Like charges repel and unlike charges attract.

In an Atom, Electron carries negative charge. The Proton carries positive charge. Since, the number of protons is equal to the number of electrons; the charges are equal and opposite. Hence, atom is neutral and carries no charge.

An electric charge occurs when the number of protons differs from the number of electrons. i.e. when there is an excess (or) deficiency of electrons, ~~occur~~ the atom is said to be charged.

The unit of charge is Coulomb.

The charge of an electron is;

$$e = -1.6 \times 10^{-19} \text{ Coulomb}$$

Voltage :

A force of attraction exists between positive and negative charges. Certain amount of energy is required to overcome the force and to move the charge. All opposite charges posses potential energy because of the separation between them. The difference in potential energy of the charge is called Potential Difference. The Potential difference is called voltage. The unit of the potential difference is volt. It is denoted by V.

If the work done in moving a charge of one coulomb between the two points is one Joule, the potential difference between the two points is one volt.

$$V = \frac{dw}{dQ} = \frac{W}{Q}$$

Where Q is the charge in Coulomb and W is the work done in Jules.

Current :

There are free electrons available in all semi conductive and conductive materials. These free electrons move random in all directions. If voltage is applied across the material, all free electrons move in one direction. This movement of electron constitutes current. It is denoted by I. The conventional direction of current is opposite to the flow of negative charges(electrons).

Current is defined as the rate of flow of charge in unit time. If one coulomb of charge is transferred in one second, the current is said to be one Ampere.

$$i = \frac{dq}{dt}; \quad I = \frac{Q}{T}$$

dq is change in charge and dt is change in time.

Resistance :

Electrical Resistance is the property of material which opposes the flow of current through it.

(i.e.) Resistance property reduces the current flow through it.

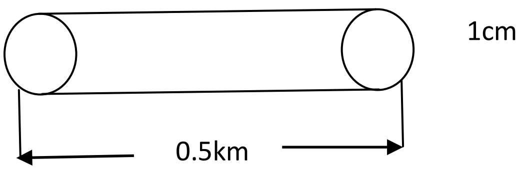
The unit of Resistance is ohm, denoted by **Ω**. The resistance of the conductor depends upon its material, length, cross sectional area and the temperature.

$$R = \frac{\rho l}{A}$$

$\boldsymbol{\rho}$ – Specific resistance of the material (ohm-meter)
l – Length of conductor (m)
A – Cross sectional area (m^2)

Example Problem:

Find the resistance of a copper wire 0.5km long and 1cm dia. Given the specific resistance of copper is 1.7×10^{-8} **Ω**-m.



Solution :

$r = \frac{d}{2} = \frac{1}{2} = 0.5 \text{ cm} = 0.5 \times 10^{-2}\text{m}$

Cross sectional area $= \pi r^2$

$= \pi \times (0.5 \times 10^{-2})^2$

$= 7.853 \times 10^{-5}$

$R = \frac{\rho l}{A}$

$= \frac{1.7 \times 10^{-8} \times 0.5 \times 10^{3}}{7.853 \times 10^{-5}}$

$= 0.1082\ \Omega$

Power and Energy :

Energy is the capacity of doing work. Energy is the work stored. It is denoted by E.

Power is the rate of change of Energy. It is denoted by P. If certain amount of energy is used over a time then

$$\boxed{\text{Power} = \frac{\text{Energy}}{\text{Time}}}$$

$$= \frac{W}{T} \quad = \frac{dw}{dt}$$

It can also be given as

$$P = \frac{W}{T} \quad = \frac{W}{Q} \times \frac{Q}{T}$$

$$\boxed{P = V \times I}$$

Energy is measured in Jules, time in seconds and power in watts.

By definition, one watt is the amount of power generated when one Jule of energy is consumed in one second.

Example problem:

What is the power in watts if energy equal to 100 J is used in 1 sec?

Solution :

$$P = \frac{\text{Energy}}{\text{Time}} \quad = \frac{100}{1} = 100 \text{ watts}$$

Circuit:

If there is a closed path exists for the flow of current, it is called circuit. The electric circuit consists of three parts.

1. Energy source
2. Load
3. Connecting wire

Example :

Consider the simple circuit shown in fig.1.1

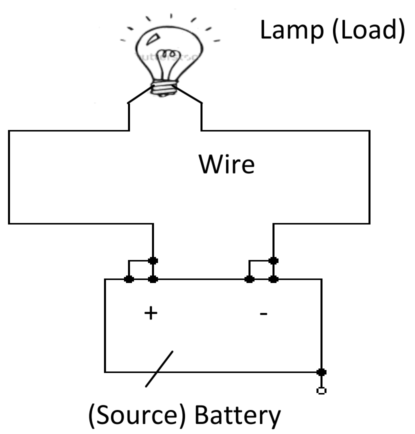


fig.1.1

A Battery is connected to a lamp with wires. There is a closed path exists from positive terminal to negative terminal of the battery through the lamp. The purpose of the circuit is to transfer energy from the source to load.

Resistor :

When the current flows through a conducting material the free electrons collide with atoms. The collisions cause the electrons to lose their energy. The loss of energy per unit charge is the drop in potential across the material. The collisions restrict the movement of electrons is called resistance and the material is called a Resistor. It is denoted by R. The unit of resistance is ohm. The symbol of Resistor is shown in fig.1.2

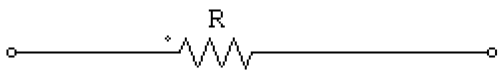


fig.1.2

1.2 OHM'S LAW

The law connecting voltage, current and resistance was established by G.S. Ohm, is known as ohm's law.

Statement :

Temperature remaining constant, the potential difference across the conductor is proportional to the current flowing through it.

$$V \propto I$$

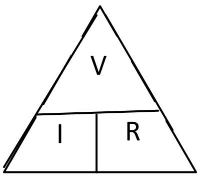
$$\boxed{V = IR}$$

Where, the constant proportionality R is the Resistance in ohms.

According to ohm's law, the current is directly proportional to voltage and inversely proportional to Resistance.

$$\boxed{I = \frac{V}{R}}$$

The fig. shows the relation between three parameters V, I and R.



Power dissipated in the Resistor

$$P = V.I$$

$$= I^2R$$

$$= \frac{V^2}{R}$$

$$\boxed{P = VI = I^2R = \frac{V^2}{R}}$$

Energy dissipated in the resistor

E = Power x time

= P x t

= I^2R t

Example Problem :

A 10Ω resistor is connected across a 20V Battery. How much current flows through the resistor ?

V = IR

$I = \frac{V}{R} = \frac{20}{10} = 2A.$

Example Problem :

Current through a 5 Ω resistor is 2A. What is the power consumed by the resistor ?

I = 2A; R = 5 Ω

Power = $I^2R = 2^2 x\ 5 = 4 \times 5 = 20$ Watts

Inductor :

When the wire is twisted into a coil, it is called an Inductor. If current is made to pass through an Inductor, electromagnetic field is formed. A change in the magnitude of current produces change in magnetic field, which induces voltage across the coil according to Faraday's laws of electromagnetic Induction.

Inductance is denoted by L. The unit of inductance is Henry.

Definition for Henry :

The Inductance is one Henry when current through the coil, changing at the rate of one Ampere per second, induces one voltage across the coil.

The symbol of Inductance is shown in fig.1.3

fig.1.3

The current voltage relation is given by

$$\boxed{V = L\frac{di}{dt}}$$

V-Voltage across the coil
L – Inductance of the coil

$\frac{di}{dt}$ – rate of change of current with time

We can rewrite the equation as

$$di = \frac{1}{L} v dt$$

Integrating both sides, we get

$$\int di = \frac{1}{L} \int vdt$$

$$\boxed{i = \frac{1}{L} \int vdt}$$

Where i is the current through the coil. The power absorbed by inductor is

$$P = V.\, I = L. \frac{di}{dt} .\, i$$

Energy accepted by Inductor is

$$P = \frac{dw}{dt}$$

$$W = \int Pdt$$

$$= \int L \,.\, i. \frac{di}{dt} .dt$$

$$= L \int i \, di$$

$$= \frac{Li^2}{2}$$

Capacitor :

Two conducting surfaces separated by an insulating medium is called capacitor. The conducting surfaces are called electrodes and the insulating medium is called dielectric.

A capacitor stores energy in the form of an electric field. The amount of charge per unit of voltage that a capacitor can store is its capacitance, denoted by C. The unit of capacitance is Farad.

By definition, one farad is the amount of capacitance when one coulomb of charge is stored with one volt across the plates.

The symbol of capacitance is shown in fig.1.4

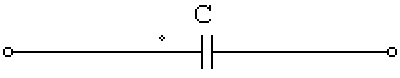


fig.1.4

Capacitance C = $\frac{Q}{V}$ = $\frac{q}{v}$

Q-charge stored in a capacitor
V-Voltage across the capacitor

We know

$i = \frac{dq}{dt}$

dq = idt

q = ∫ idt

we can rewrite the equation

$C = \frac{q}{v}$; $V = \frac{q}{c}$

$$\boxed{V = \frac{1}{C} \int idt}$$

From the above equation, current through the capacitor is

$V = \frac{1}{C} \int idt$

$CV = \int idt$

Differentiating on both sides,

$$\boxed{C.\frac{dv}{dt} = i}$$

Power absorbed by the capacitor is given by

P = VI

$P = V.\ C\frac{dv}{dt}$

Energy stored in the capacitor is

$W = \int Pdt$

$= \int V . C\frac{dv}{dt}\ dt$

$= C \int V.dv$

$$\boxed{W = \frac{cv^2}{2}}$$

1.3 KIRCHHOFF'S LAWS

Kirchhoff stated two laws. One for currents meeting at any junction in the circuit and other for voltage around closed loop.

Kirchhoff's Voltage law :

Statement :

The algebraic sum of the voltages around any closed path in the circuit is equal to zero.

Potential rise = Potential drop

Consider the circuit shown in fig.1.5

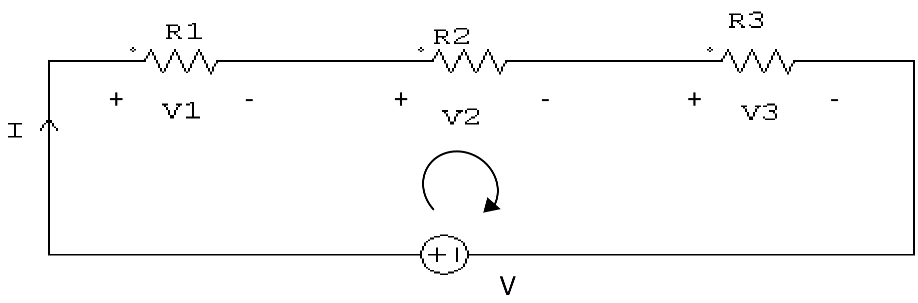


fig.1.5

Current I leaves the positive terminal of the battery, and passes through resistors $R_{1,}$ R_2 & R_3 and enters into the negative terminal. As the current passes in the circuit, the sum of the voltage drops around the loop is equal to the supply voltage.

Let the voltage drop across $R_{1,}$ R_2 & R_3 as $V_{1,}$ V_2 & V_3 . Supply voltage is V.

$V_1 = IR_1$; $V_2 = IR_2$; $V_3 = IR_3$

$$\boxed{V = V_1 + V_2 + V_3}$$

$$= IR_1 + IR_2 + IR_3$$

$$= I(R_1 + R_2 + R_3)$$

$$I = \frac{V}{R1 + R2 + R3}$$

Example Problem :

For the circuit shown in figure, Validate the Kirchhoff's voltage law.

fig.1.6

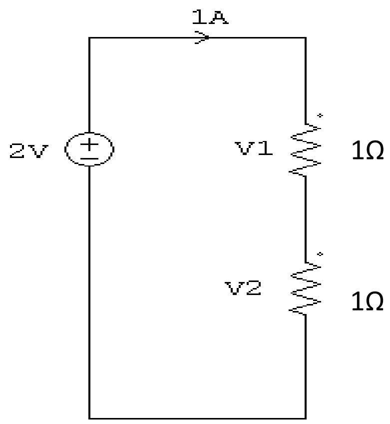


fig.1.6

$V_1 = IR_1 = 1A \times 1\Omega = 1V$

$V_2 = IR_2 = 1A \times 1\ \Omega = 1V$

According to Kirchhoff's voltage law,

$V_1 + V_2 = 1 + 1 = 2\ V\ = V$

Example Problem:

Find the current flowing through the given circuit fig.1.7

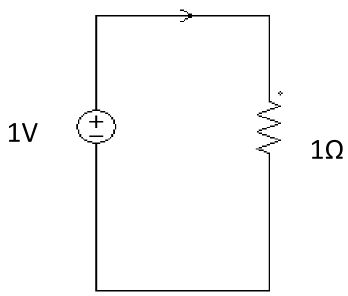


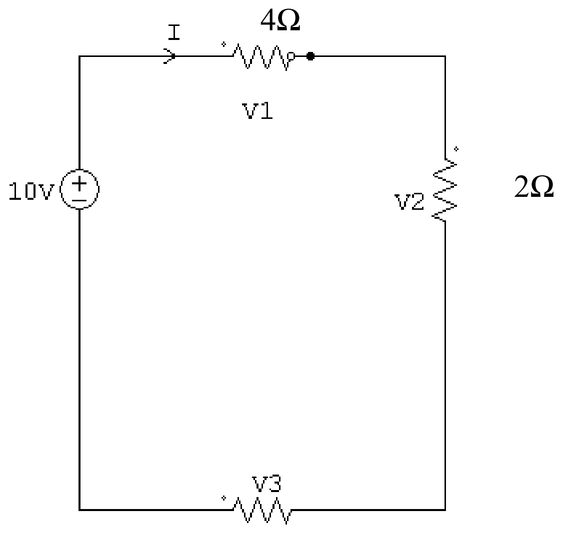
fig.1.7

According to ohm's law;

$I = \frac{V}{R} = \frac{1V}{1\Omega} = 1A$

Example Problem:

Find the current in the circuit shown in fig.1.8. Determine the voltage drop across each Resistor.



4Ω

Fig 1.8

Let the voltage drop across the Resistors are $V_{1,}$ V_2 & V_3

By applying ohm's law, we can write $V_1 = 4I$; $V_2 = 2I$; $V_3 = 4I$

By applying Kirchhoff's voltage law, we can write

$$V = V_1 + V_2 + V_3$$

$$10 = 4I + 2I + 4I$$

$$10 = 10\, I$$

$$I = \frac{10}{10} = 1A$$

Voltage drop across each Resistor

$$V_1 = 4I = 4V$$

$$V_2 = 2I = 2V$$

$$V_3 = 4I = 4V$$

Kirchhoff's current law :

Statement :

Sum of the currents entering into any junction is equal to the sum of the currents leaving that junction. For example consider the circuit shown in Fig 1.9.

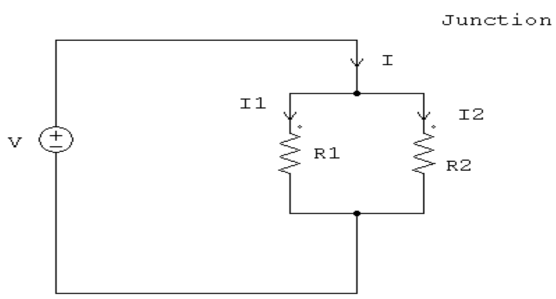


Fig 1.9

I is the current entering into the junction. I_1 and I_2 are currents leaving that junction. According to Kirchhoff's current law,

$$\boxed{I = I_1 + I_2}$$

In general Kirchhoff's current law states that the algebraic sum of currents entering into the junction and currents leaving that junction is equal to zero as shown in Fig 1.10

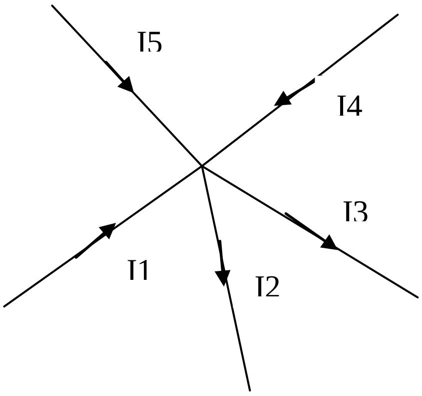


Fig 1.10

$I_1 - I_2 - I_3 + I_4 + I_5 = 0$

$I_1 + I_4 + I_5 = I_2 + I_3$

Example Problem:

Find the current I in the given circuit.

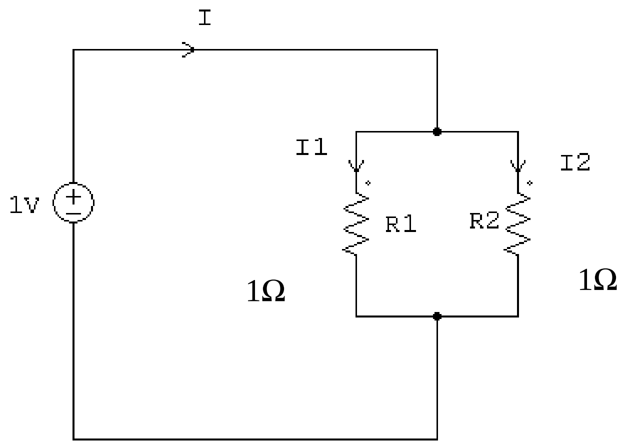


Fig 1.11

By applying ohm's law

$$I_1 = \frac{V}{R1} = \frac{1V}{1\Omega} = 1A$$

$$I_2 = \frac{V}{R2} = \frac{1V}{1\Omega} = 1A$$

According to Kirchhoff's current law,

$I = I_1 + I_2$

$= 1A + 1A$

$= 2A$

Current division :

In Parallel circuit current divides and flows in all branches. The total current entering into the circuit is divided into the branches according to the resistance values. The branch having higher resistance allows lesser current, and the branch with lower resistance allows more current. Consider the circuit in Fig.1.12.

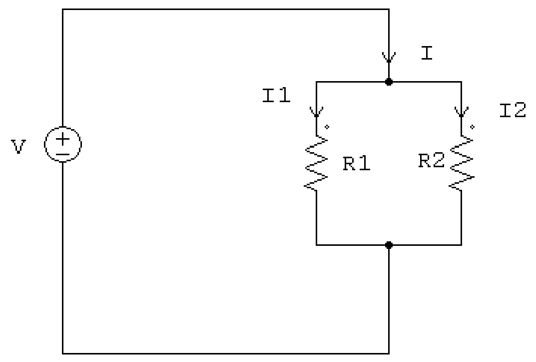


Fig 1.12

According to Kirchhoff's Current Law,

$$I = I_1 + I_2$$

$$= \frac{V}{R1} + \frac{V}{R2}$$

$$\frac{1}{R} = \frac{I}{V} = \frac{R1+R2}{R1R2}$$

$$R = \frac{R1R2}{R1+R2} \quad \text{———————} 1$$

R - Total Resistance

Total current

$$I = \frac{V}{R} = \frac{V(R1+R2)}{R1R2} \quad \text{———————} 2$$

According to KVL, we can write

$$V = I_1 R_1$$

substituting in Eq2

$$I = R_1 + R_2$$

$$\boxed{I_1 = \left(\frac{R2}{R1+R2}\right) . I}$$

similarly, we can prove

$$\boxed{I_2 = \left(\frac{R1}{R1+R2}\right) . I}$$

Example Problem:

Find the current I_1 and I_2 in the given circuit in Fig 1.13

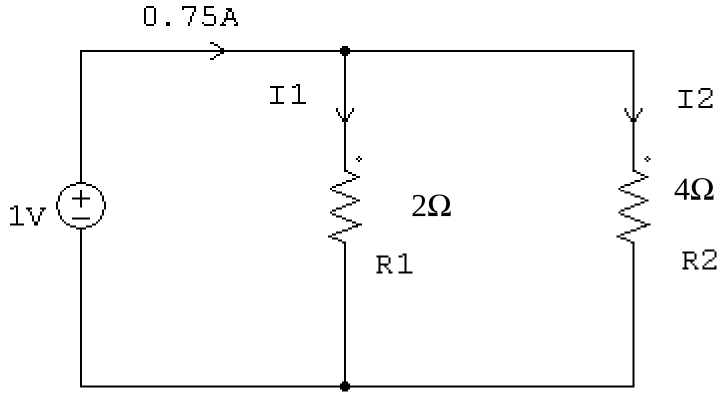


Fig 1.13

$$I_1 = \left(\frac{R2}{R1+R2}\right) X\ I$$

$$= \frac{4}{6}\ x\ 0.75 = 0.5A$$

$$I_2 = \frac{R1}{R1+R2} X\ I$$

$$= \frac{2}{6}\ x\ 0.75 = 0.25A$$

$R_1 < R_2$ hence $I_1 > I_2$

Kirchhoff's current law is verified as

$$I = I_1 + I_2$$

$$0.75 = 0.5 + 0.25$$

1.4 RESISTANCE CONNECTED IN SERIES :

When the Resistors are connected together end to end so that the same current passes from one to other, they are said to be connected in series. Consider the circuit shown in Fig 1.14

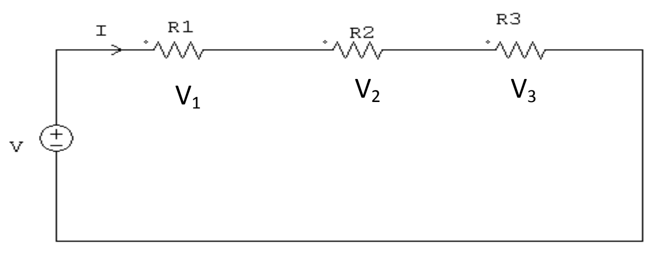


Fig 1.14

According to Kirchhoff's voltage law, we can write

$$V = V_1 + V_2 + V_3$$

$$V = IR_1 + IR_2 + IR_3$$

$$V = I(R_1 + R_2 + R_3)$$

$$\boxed{R = \frac{V}{I} = R_1 + R_2 + R_3}$$

R is the equivalent Resistance

Hence, the circuit can be redrawn as

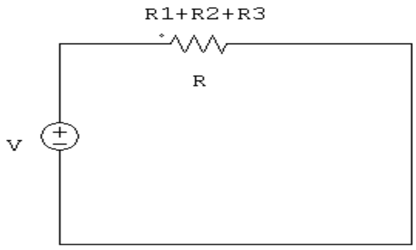


Fig 1.15

1.5 RESISTANCE CONNECTED IN PARALLEL :

When the resistances are connected across one another, so that the same voltage is applied to each resistance, then they are said to be in parallel. Consider the circuit shown in Fig 1.16

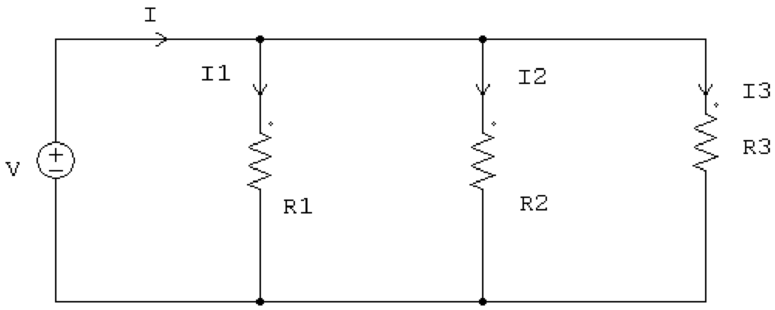


Fig 1.16

According to Kirchhoff's current law, we can write,

$$I = I_1 + I_2 + I_3$$

$$= \frac{V}{R1} + \frac{V}{R2} + \frac{V}{R3}$$

$$= V\left[\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}\right]$$

$$\frac{1}{R} = \frac{1}{V} = \frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}$$

$$R = \frac{R1R2+R2R3+R3R1}{R1R2R3}$$

R is the Total equivalent Resistance

1.6 RESISTANCE CONNECTED IN SERIES & PARALLEL :

Example : Find the equivalent Resistance for the circuits shown in Fig.1.17

(i)

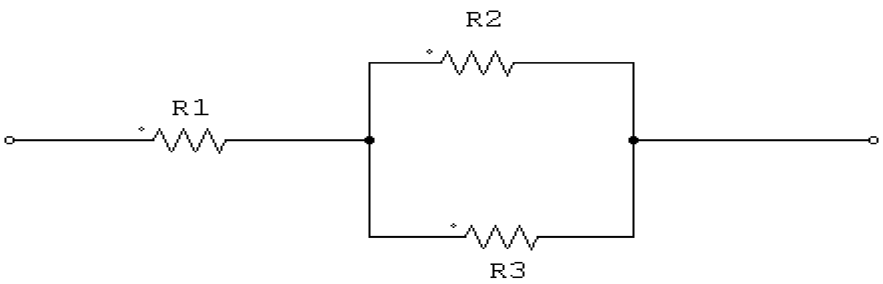


Fig.1.17

Solution :
R $_2$& R_3 are in parallel hence it can be written as $R_2 R_3 / R_2 + R_3$

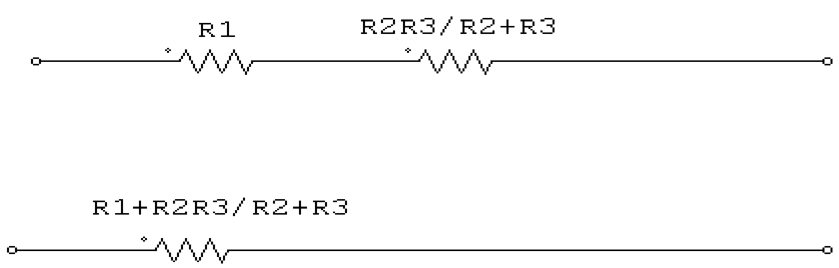


Fig.1.18

Example Problem:

Find the equivalent resistance between A & B in Fig.1.19

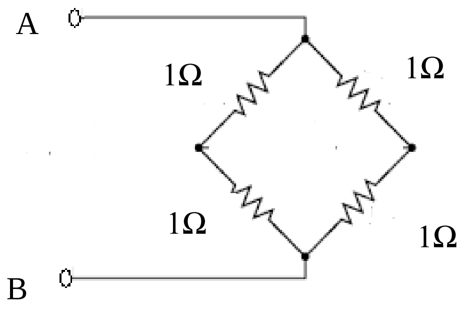


Fig.1.19

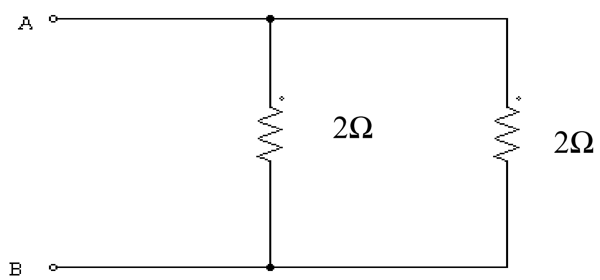
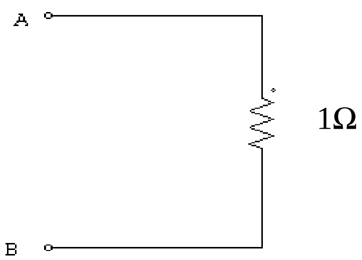


Fig.1.20



$\left(\frac{2X2}{2+2} = \frac{4}{4} = 1\Omega\right)$

Fig.1.21

Exercise Problem:

Find the current through the 10Ω Resistor and the power consumed by the 10Ω Resistor in Fig.1.22.

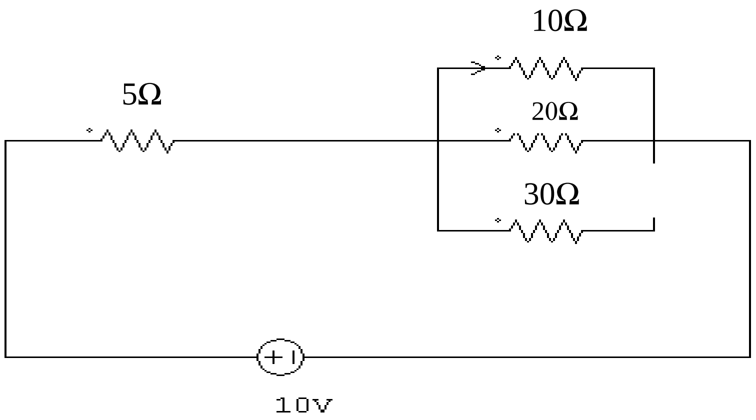


Fig.1.22

Example Problem:

Find the total Resistance between A & B in Fig.1.23.

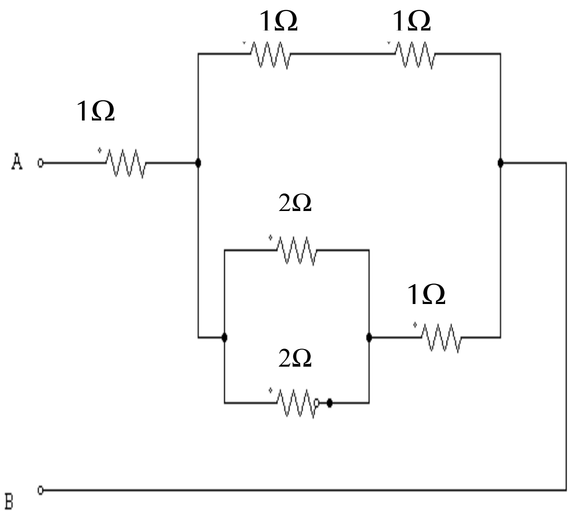


Fig.1.23

We can redraw the circuit by

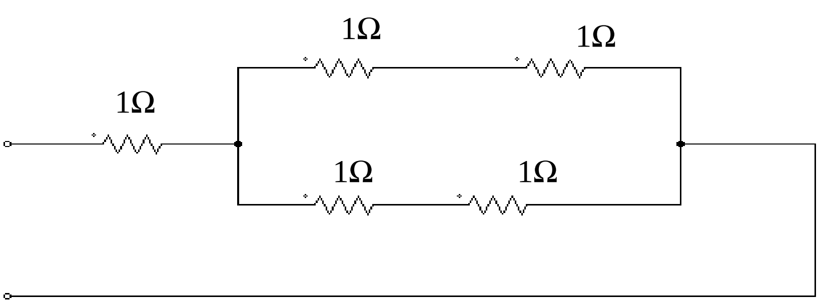


Fig.1.24

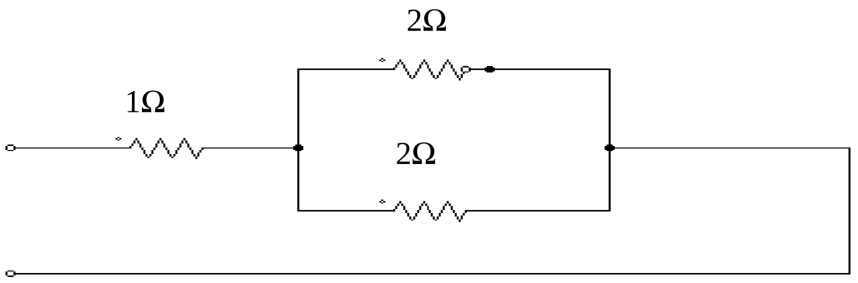


Fig.1.25

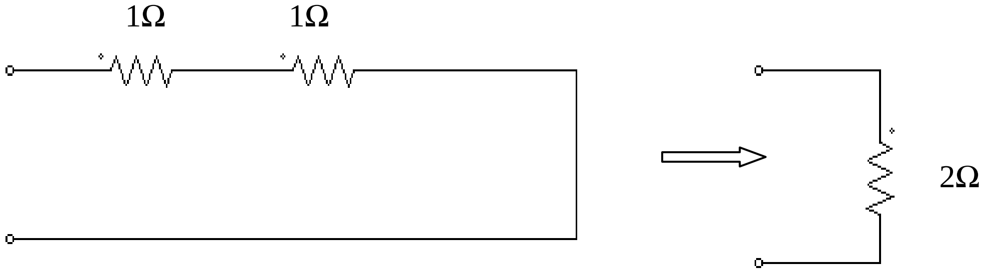


Fig.1.26

Answer: Total Resistance between A & B is 2Ω

Example Problem:

Determine the total current in the given circuit in Fig.1.27.

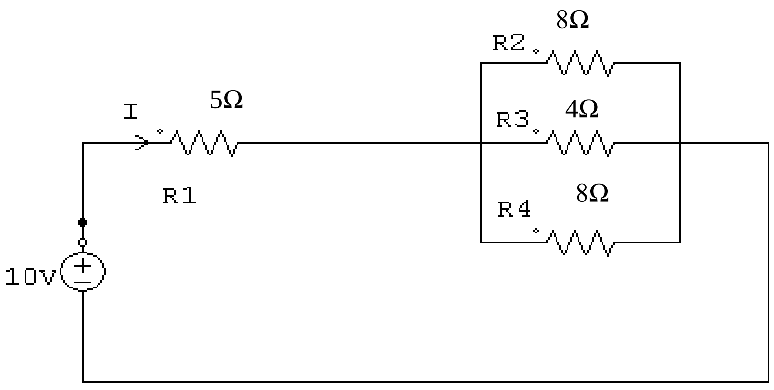


Fig.1.27

Solution :

Resistances R_2, R_3 and R_4 are in parallel Let the Equivalent resistance be R_5

$$R_5 = \frac{1}{R2} + \frac{1}{R3} + \frac{1}{R4}$$

$$R_5 = \frac{R2R3+R3R4+R2R4}{R2R3R4}$$

$R_5 = \frac{8X4+4X8+8X8}{8X4X8}$

$R_5 = \frac{32+32+64}{256}$ $= 0.5\ \Omega$

R_1 & R_5 are in series

Hence total resistance = $R_1 + R_5$

$= 5 + 0.5 = 5.5\Omega$

Total current I = V/R

=10/5.5

=1.8A

$\boxed{I = 1.8A}$

Example Problem:

Find the current I_L through the 6Ω Resistor and calculate the power consumed by 6Ω Resistor in Fig.1.28.

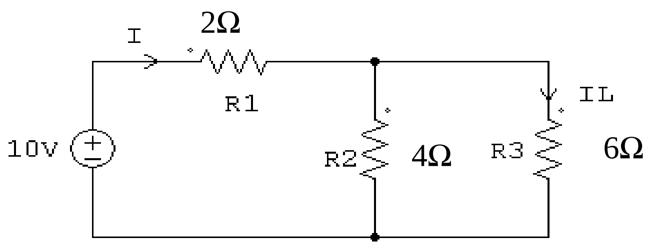


Fig.1.28

R_2 & R_3 are in parallel

$R_2 \|^1\ R_3 = \frac{R2R3}{R2+R3} = \frac{4X6}{6+4} = 2.4\Omega$

Total equivalent resistance $R_{eq} = R_1 + R_2 \|^1 R_3$

$= 2\ \Omega + 2.4\ \Omega$

$= 4.4\ \Omega$

Total current $I = \frac{V}{Req} = \frac{10}{4.4} = 2.27A$

Current through 6Ω Resistor

$I_L = \frac{R2}{R2+R3}$ X I

$= \frac{4}{6+4}$ X 2.27

$\boxed{I_L = 0.908A}$

Power consumed by 6Ω Resistor

$P = I^2 R$

$= (0.908)^2 \times 6$

$$\boxed{P = 4.95W}$$

Example Problem :

Find the current I_L through the load Resistance R_L in Fig.1.29.

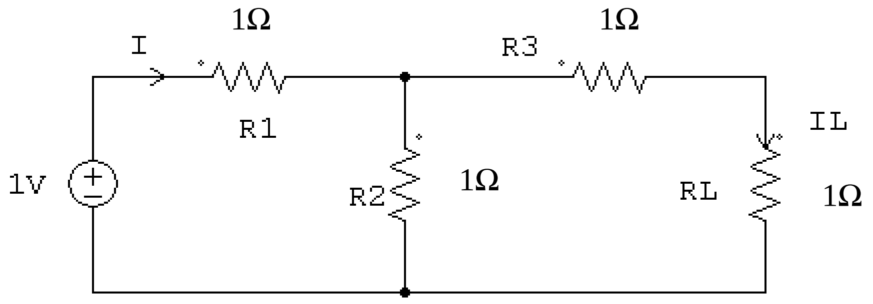


Fig.1.29

To find the total equivalent resistance reduce the resistance from the load side towards the source. R_3 and R_L are in series. Hence it is reduced as $R_3 + R_L$

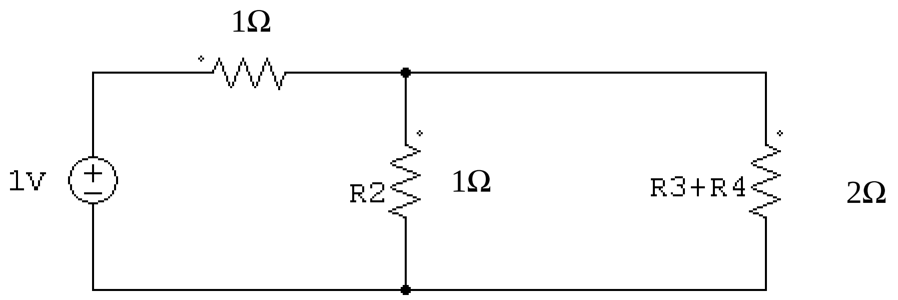
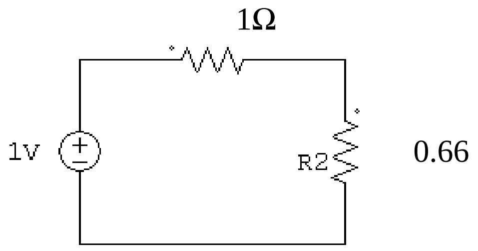


Fig.1.30

1 Ω & 2 Ω resistance are in parallel



$$\frac{1 \times 2}{1+2} = 0.66\ \Omega$$

Fig.1.31

1Ω and 0.66Ω are in series

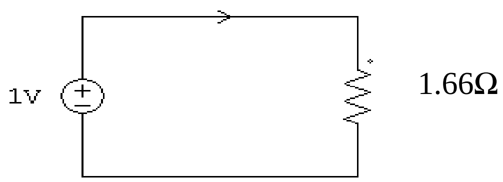


Fig.1.32

Total Current $I = \frac{V}{R} = \frac{1}{1.66} = 0.6$ A

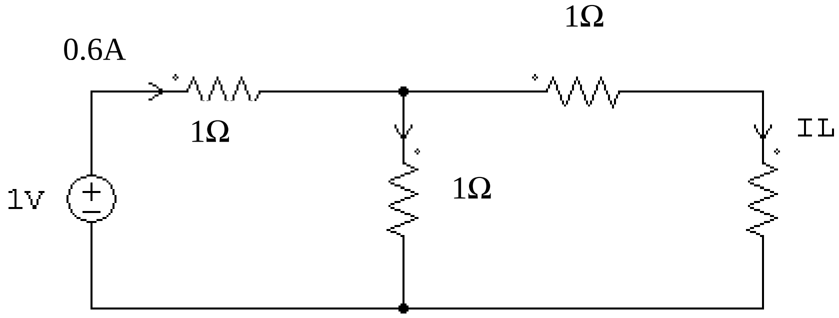


Fig.1.33

$I_L = \frac{1}{3} X 0.6 = 0.2$ A

Example Problem:

Find total current, resistance and voltage drop across each resistances in the following circuit.

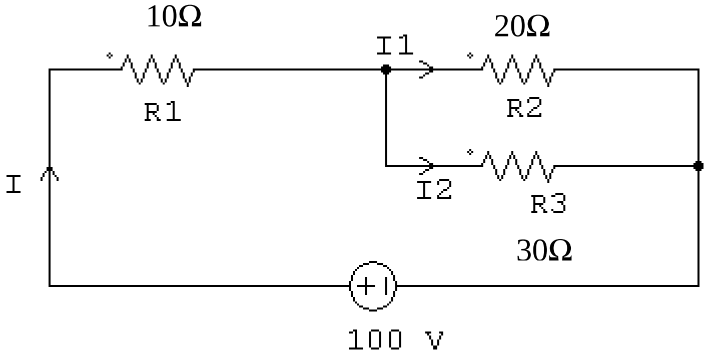


Fig.1.34

Total Resistance $= \frac{20 X 30}{20+30} + 10$

$R_{eq} = 22\Omega$

Total current $I = \frac{V}{Req} = \frac{100V}{22\Omega} = 4.54A$

Current through 20Ω Resistance

$I_1 = \frac{R3}{R2+R3} X I$

$= \frac{30}{50}$ X 4.55 = 2.73A

current through 30Ω Resistance is

$I_2 = \frac{R2}{R2+R3} X I$

$= \frac{20}{30+20}$ X 4.55 = 1.82A

Voltage drop across $R_1 = IR_1$

= 4.54 X 10
= 45.4V

Voltage drop across $R_2 = I_1R_2$

= 2.73A X 20Ω
= 54.6 V

Voltage drop across $R_3 = I_2R_3$

= 1.82A X 30Ω
= 54.6 V

Example Problem:

Three Resistors 2Ω, 4Ω, 10Ω are connected in parallel across 12V battery in Fig.1.35. Find the current through each resistor. Also find the power dissipated in each resistor.

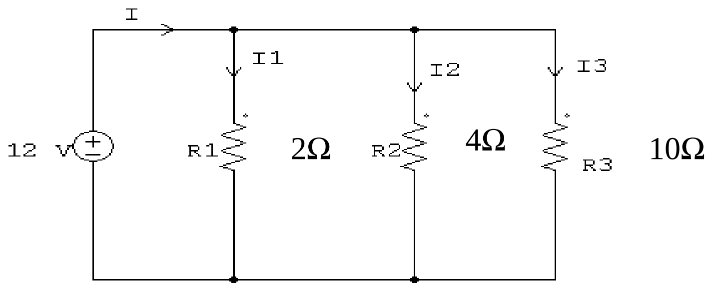


Fig.1.35

Total Resistance

$$\frac{1}{RT} = \frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}$$

$$R_T = \frac{R1R2+R2R3+R1R3}{R1R2R3} = \frac{8+40+20}{2 X 4 X 10} = 0.85\ \Omega$$

Total Current I $= \frac{V}{RT} = \frac{12V}{0.85\Omega}$ = 14.12A

Current flowing through 2Ω Resistor

$$I_1 = \frac{2.86\Omega}{4.86\Omega} \text{ X } 14.12A = 8.31A$$

Current flowing through 4Ω Resistor\

$$I_2 = \frac{1.66\Omega}{5.66\Omega} \text{ X } 14.12A = 4.14A$$

Current flowing through 10Ω Resistor

$$I_3 = \frac{1.33\Omega}{11.33\Omega} \text{ X } 14.12A = 1.657A$$

Power dissipated in R_1

$$P_1 = I_1^2 R_1$$

$$= (8.31)^2 \text{ X } 2 = 138.11W$$

Power dissipated in R_2

$$P_2 = I_2^2 R_2$$

$$= (4.14)^2 \text{ X } 4 = 68.56W$$

Power dissipated in R_3

$$P_3 = I_3^2 R_3$$

$$= (1.657)^2 \text{ X } 10\Omega \quad = 27.46W$$

Mesh current method :

Loop is a closed path. Mesh is defined as a loop which does not contain any other loops within it. Mesh analysis is applied to the network which has one or many voltage sources and it does not have any feedback.

Method:

Step1 : Confirm that circuit does not have any feed back

Step2 : Assign mesh currents.

Step3 : Write Kirchhoff's voltage law equation for each mesh.

Step4 : Solve the equation to find unknown.

Consider the circuit shown in figure 1.36

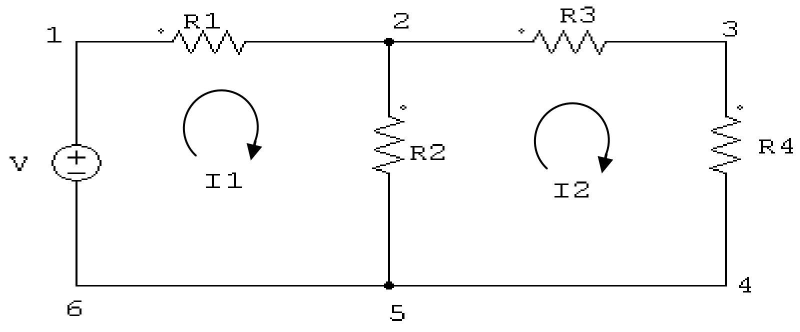


Fig.1.36

Step1: There is no feed back.

Step2: There are two mesh 12561 and 23452 in the network. Let us assign mesh currents I_1 and I_2 with directions as indicated in the figure.

Step3: Considering the mesh 12561 alone we observe that current I_1 is passing through R_1 and I_1- I_2 is passing through R_2. By applying Kirchhoff's voltage law, we can write

$$V = I_1 R_1 + (I_1 - I_2)R_2$$

$$V = I_1(R_1 + R_2) - I_2 R_2 \text{ ---------------------1}$$

Similarly consider the mesh 23452, current I_2 is passing through R_3 and R_4 and $(I_2 - I_1)$ is passing through R_2. By applying Kirchhoff's voltage law, we can write,

$$R_2(I_2 - I_1) + R_3I_2 + R_4I_2 = 0$$

$$I_2(R_2 + R_3 + R_4) - I_1 R_2 = 0 \text{ ---------------------2}$$

By solving the above equations 1 & 2 we can find currents I_1 & I_2

Example Problem:

Find the current flowing through each resistor using mesh current method for the circuit shown in figure 1.37

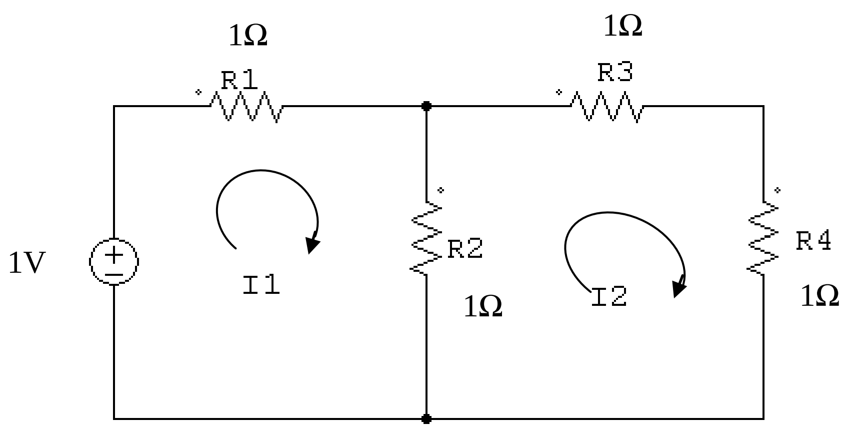


Fig.1.37

The mesh equations are,

$1V = 1.I_1 + 1.(I_1-I_2)$
$O = 1.(I_2-I_1) + (1I_2 + 1I_2)$
We can rearrange the above equations as

$1 \ = 2I_1 - I_2$

$0 = \ - I_1 + 3I_2$

The above equations can be written in Matrix form as follows .

$$\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}I1\\I2\end{bmatrix} \begin{bmatrix}2 & -1\\-1 & 3\end{bmatrix}$$

V = I R

We can solve this matrix equation by applying Grammer's rule .

$$I_1 = \frac{\begin{vmatrix}1 & -1\\0 & 3\end{vmatrix}}{\begin{vmatrix}2 & -1\\-1 & 3\end{vmatrix}}$$

$= 3/5 = 0.6A$

$$I_2 = \frac{\begin{vmatrix}2 & 1\\-1 & 0\end{vmatrix}}{\begin{vmatrix}2 & -1\\-1 & 3\end{vmatrix}}$$

$= 1/5 = 0.2A$

Current flowing through $R_1 = I_1 = 0.6A$
Current flowing through $R_2 = |I_1 - I_2| = 0.4A$
Current flowing through $R_3 = I_2 = 0.2A$
Current flowing through $R_4 = I_4 = 0.2A$
Find the current 'I' using mesh current analysis for the given circuit

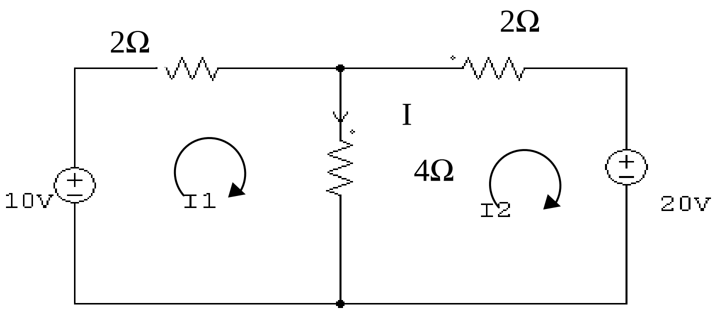


Fig.1.38

The mesh equations are

In the mesh 1

$10 - 2I_1 - 4(I_1 - I_2) = 0$

$10 - 6I_1 + 4I_2 = 0$

$10 = 6I_1 - 4I_2$ ———— 1

In the mesh 2

$-4(I_2 - I_1) - 2I_2 - 20 = 0$

$-4I_2 + 4I_1 - 2I_2 - 20 = 0$

$-20 = -4I_1 + 6I_2$ ———— 2

Equation 1 & 2 can be written in matrix form as follows.

$$\begin{bmatrix} 10 \\ -20 \end{bmatrix} = \begin{bmatrix} I1 \\ I2 \end{bmatrix} \begin{bmatrix} 6 & -4 \\ -4 & 6 \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} 10 & -4 \\ -20 & 6 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ -4 & 6 \end{vmatrix}}$$

$$I_1 = \frac{60-80}{20} = \frac{-20}{20} = -1A$$

$$I_2 = \frac{\begin{vmatrix} 6 & 10 \\ -4 & -20 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ -4 & 6 \end{vmatrix}}$$

$$I_2 = \frac{-120+40}{20} = \frac{-80}{20} = -4A$$

Hence

$I = I_1 - I_2$

$= -1 - (-4) = -1 + 4 = 3A$

Example Problem:

Find the load current and power delivered to the load using mesh current method for the circuit shown in Fig.1.39

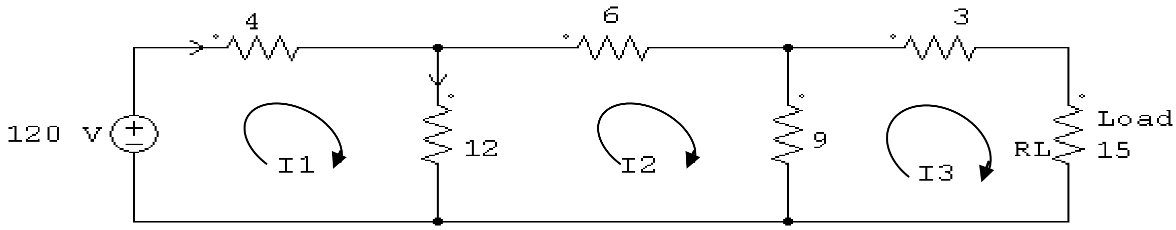


Fig.1.39

The mesh equations are,

In mesh 1

$I20 -4I_1 – 12(I_1\text{-}I_2) = 0$

$120=16\ I_1 – 12\ I_2$ ---------------------1

In mesh 2

$-12(I_{2\text{-}}\ I_1) - 6\ I_2\ \ - 9(I_{2\text{-}}\ I_3) = 0$

$-12\ I_2 +12\ I_1\text{-}6\ I_2\text{-}9\ I_2\text{+}9\ I_3\text{=}0$

$0 = 12\ I_1\text{-}27\ I_2\text{+}9\ I_3$ ------------------2

In mesh 3

$-9(I_3\text{-}\ I_2\ )\text{-}3\ I_3\text{-}15\ I_3 = 0$

$-9\ I_3\text{+}9\ I_2\ \text{-}3\ I_3\text{-}15\ I_3 = 0$

$9\ I_2\text{-}27\ I_3 = 0$ ----------------------------3

Equations 1,2 & 3 can be written in matrix form

$$\begin{vmatrix} 120 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} I1 \\ I2 \\ I3 \end{vmatrix} \left| \begin{bmatrix} 16 & -12 & 0 \\ 12 & -27 & 9 \\ 0 & 9 & -27 \end{bmatrix} \right|$$

$$I_3 = \frac{\begin{bmatrix} 16 & -12 & 120 \\ 12 & -27 & 0 \\ 0 & 9 & 0 \end{bmatrix}}{\begin{bmatrix} 16 & -12 & 0 \\ 12 & -27 & 9 \\ 0 & 9 & -27 \end{bmatrix}}$$

$$= \frac{12960}{6480}$$

= 2A

power dissipated in the load

$$P = I^2 RL$$
$$= 2^2 \times 15$$
$$=60 \text{ watts}$$

Mesh equations by Inspections method

The mesh equations can be written in matrix form by inspection without going through detailed step.

consider the circuit shown in Fig.1.40

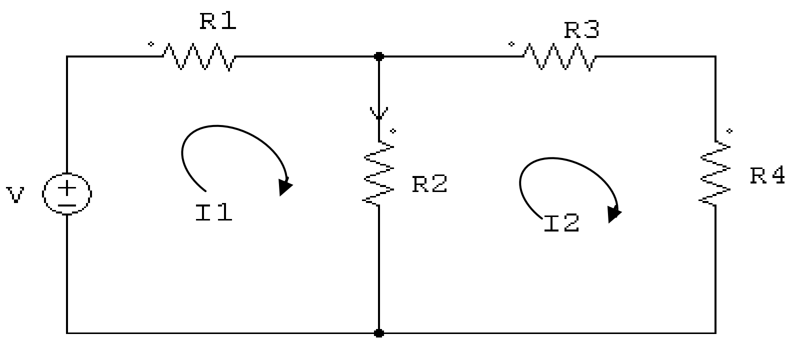


Fig.1.40

$$\begin{vmatrix} V \\ 0 \end{vmatrix} = \begin{vmatrix} I1 \\ I2 \end{vmatrix} \begin{vmatrix} R11 & R12 \\ R21 & R22 \end{vmatrix}$$

R_{11} – Self Resistances in mesh 1
R_{22} – Self Resistances in mesh 2
R_{12} – Mutual Resistance between mesh 1 & mesh 2
R_{21} – Mutual Resistances between mesh 2 & mesh 1

Hence,

$$\begin{vmatrix} V \\ 0 \end{vmatrix} = \begin{vmatrix} I1 \\ I2 \end{vmatrix} \begin{vmatrix} R1+R2 & -R12 \\ -R2 & R2+R3+R4 \end{vmatrix}$$

Since, mesh currents I_1 & I_2 flows in opposite direction through R_2, it is given –ve sign.

Example Problem:

Find the current I through 20 Ω resistance in the given circuit in Fig.1.41.

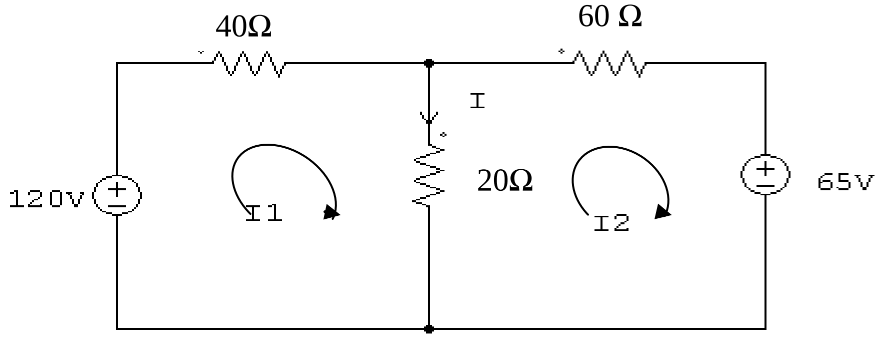


Fig.1.41

Mesh equations in the matrix form can be written by inspecting the circuit

$$\begin{vmatrix} 120 \\ -65 \end{vmatrix} = \begin{vmatrix} I1 \\ I2 \end{vmatrix} \begin{vmatrix} 60 & -20 \\ -20 & 80 \end{vmatrix}$$

$$I_1 = \frac{\begin{vmatrix} 120 & -20 \\ -65 & 80 \end{vmatrix}}{\begin{vmatrix} 60 & -20 \\ -20 & 80 \end{vmatrix}} = 1.89A$$

$$I_2 = \frac{\begin{vmatrix} 60 & 120 \\ -20 & -65 \end{vmatrix}}{\begin{vmatrix} 60 & -20 \\ -20 & 80 \end{vmatrix}} = -0.34A$$

Current through 20 resistor is

$$\begin{aligned} I &= I_1 - I_2 \\ &= 1.89 + 0.34 \\ &= 2.23A \end{aligned}$$

1.7-THEVENIN'S THEOREM

Statement :

Thevenin's theorem states that any circuit having a number of voltage sources, resistances can be replaced by a simple equivalent circuit consisting of a single voltage source in series with resistance, where the value of voltage source is equal to the open circuit voltage across the output terminals, and resistance is equal to the resistance seen into the network across the output terminals, as shown in Fig.1.42.

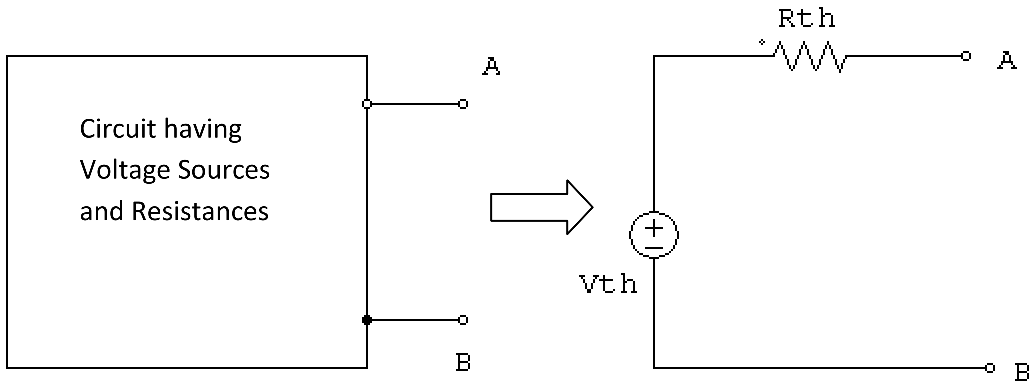


Fig.1.42

V_{th} – Open circuit voltage across the output terminals

R_{th} – Total equivalent Resistance seen into the network.

Example:

Consider the given circuit shown in Fig.1.43

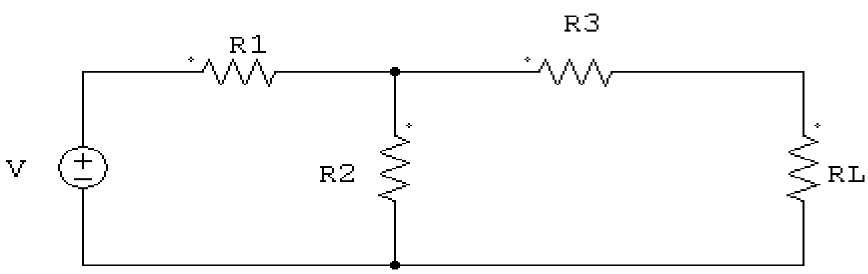


Fig.1.43

Let us find the current through R_L.

By applying Thevenin's Theorem, the given circuit can be replaced by the Thevenin's equivalent circuit shown in Fig.1.44

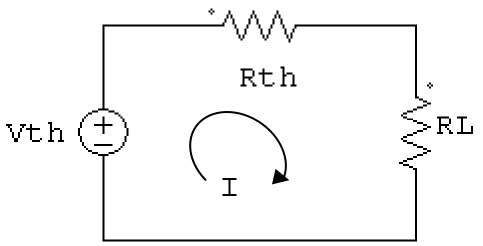


Fig.1.44

Current through load Resistance I = $\frac{V}{Rth+RL}$

Method :

Step 1: Find Thevenin's equivalent resistance R_{th} for the given circuit. To find R_{th} open the load resistance and replace the voltage source by its internal resistance (or) replace by short if it's internal resistance is zero. Circuit is redrawn as in Fig.1.45 to find R_{th}.

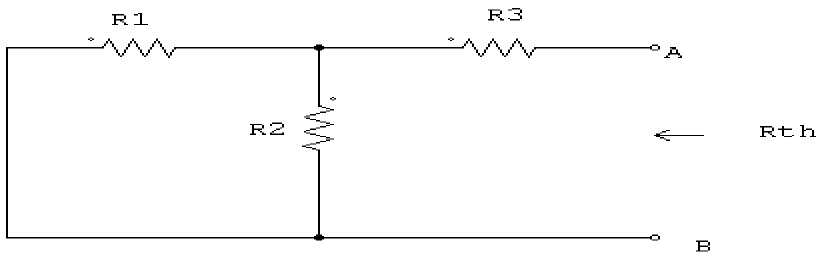


Fig.1.45

$R_{th} = R_1 || R_2 + R_3$

$= \frac{R1R2}{R1+R2} + R_3$

$= \frac{R_1 R_2 + R_3 (R_1+R_2)}{R_1 + R_2}$

$$\boxed{R_{th} = \frac{R1R2+R1R3+R2R3}{R1+R2}}$$

Step 2: Find open circuit voltage V_{th}. To find V_{th}, open the load resistance and find the voltage across the output terminals. Circuit is redrawn as follows in Fig.1.46.

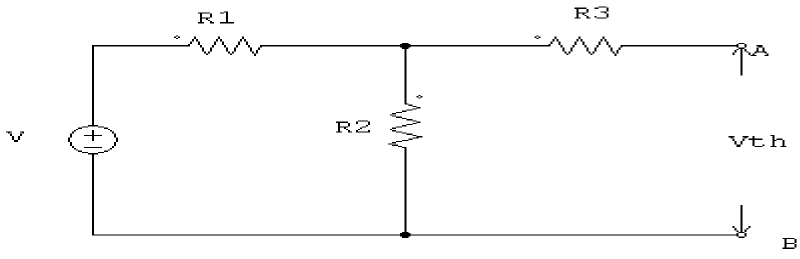


Fig.1.46

$$V_{th} = \frac{R2}{R1+R2} \, .V$$

Step 3: Draw Thevenin's equivalent circuit and find current through load resistance.

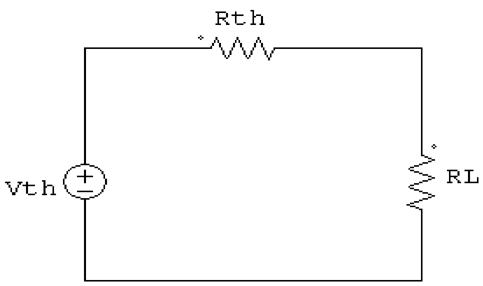


Fig.1.47

Current flowing through load resistance

$$\boxed{I = \frac{Vth}{Rth+RL}}$$

Example Problem :

Find the current I through load resistance in Fig.1.48. using Thevenin's Theorem :

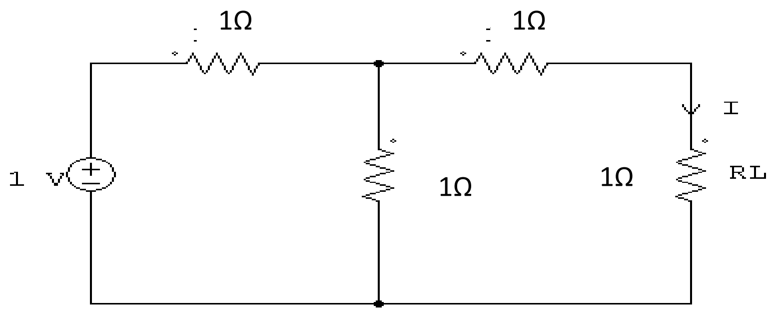


Fig.1.48

Step 1: To find Thevenin's Resistance R_{th} open the load resistance and Voltage source is replaced by short and circuit is redrawn as follows in Fig.1.49.

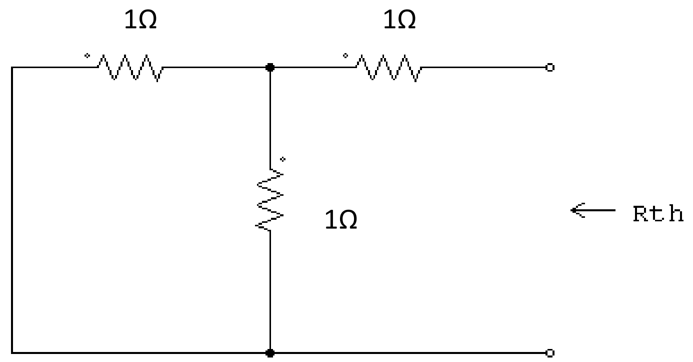


Fig.1.49

$$R_{th} = \frac{1}{2} + 1 = 1.5\ \Omega$$

Step 2: To find the open circuit voltage V_{th} , open the load resistance and find the voltage across the terminals

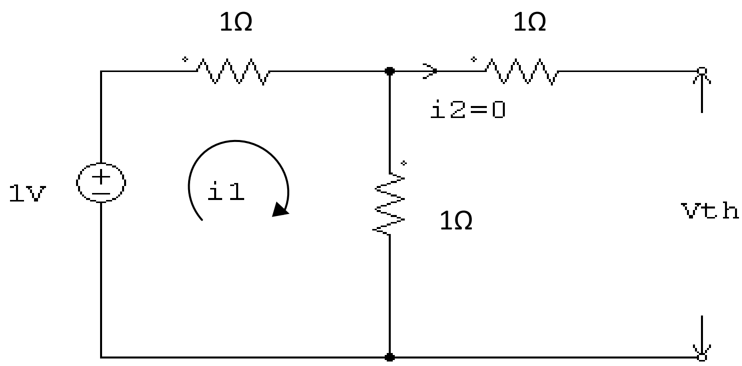


Fig.1.50

$$i_1 = \frac{1}{1+1} = 0.5\ A$$

$$V_{th} = 0.5 \times 1\ \Omega = 0.5V$$

Step 3: Thevenin's equivalent circuit is drawn as follows.

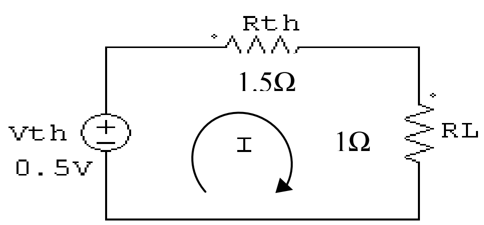


Fig.1.51

$$I = \frac{Vth}{Rth+RL} = \frac{0.5V}{1.5\Omega+1\Omega} = \frac{0.5}{2.5}$$

$$\boxed{I = 0.2A}$$

Example Problem:

Find the current through 4 Ω resistor using Thevenin's theorem

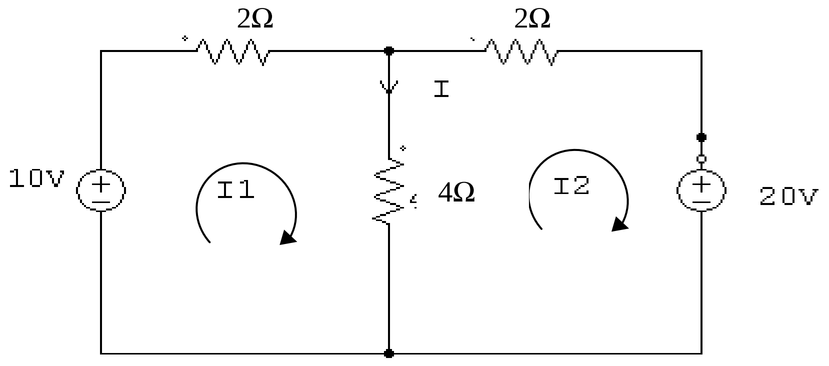


Fig.1.52

Step 1: To find Thevenin's Resistance R_{th}, open the load resistance & voltage source is replaced by short and the circuit is redrawn as follows

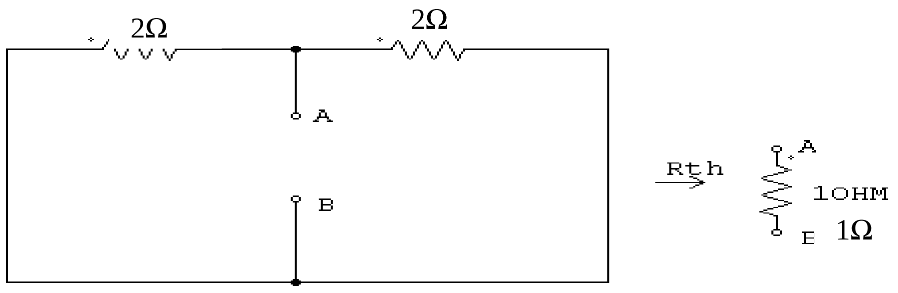


Fig.1.53

Step 2: To find the open circuit voltage v_{th} open the load resistance and find the voltage across the terminals AB circuit is redrawn as follows.

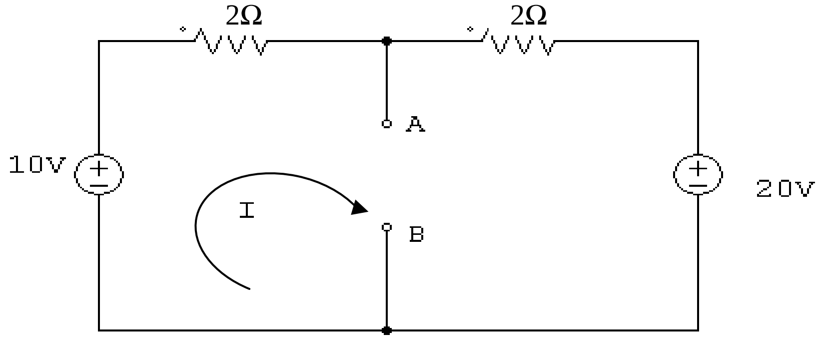


Fig.1.54

Applying Kirchoff's Voltage law,

We can write

$$10 - 2I - 2I - 20 = 0$$
$$-4I = 10$$
$$I = -2.5A$$

'-' sign indicates the direction of current. Current flows in anticlockwise direction from 20v source towards 10v source.

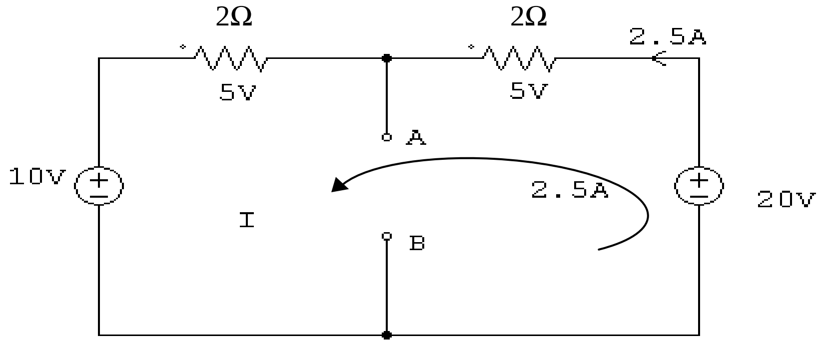


Fig.1.55

Open circuit voltage v_{th} between the terminals A & B is

$V_{th} = 20V - 5V = 15V$

Step 3: Thevenin's equivalent circuit is drawn in the following circuit.

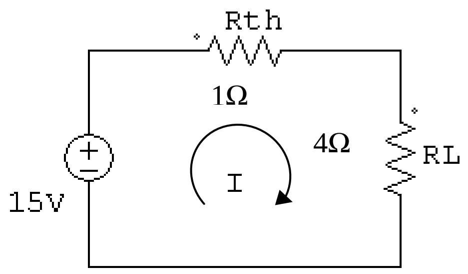


Fig.1.56

$I = \frac{Vth}{Rth+RL} = \frac{15V}{5\Omega} = 3A$

Example Problem:

Find the current I flowing through the load resistance R_L using Thevenin's theorem for the circuit shown in Fig.1.57

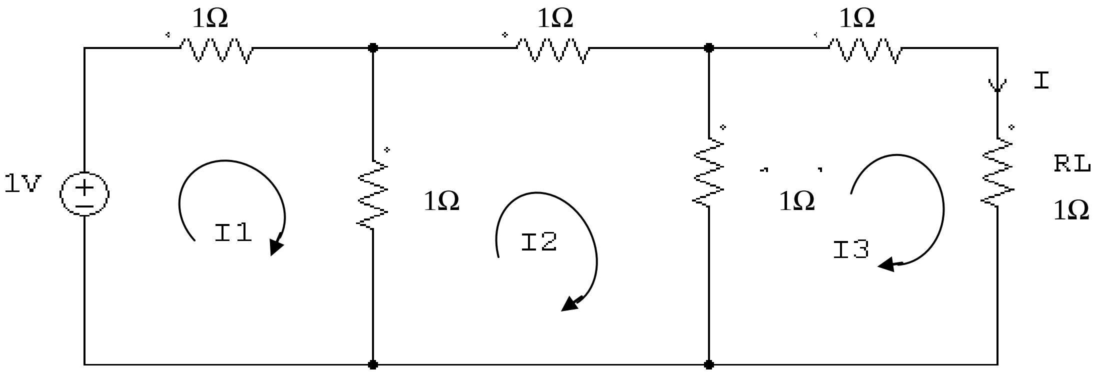


Fig.1.57

Step 1: To find Thevenin's resistance R_{th} open the load resistance & voltage source is replaced by short circuit and the circuit is redrawn as follows.

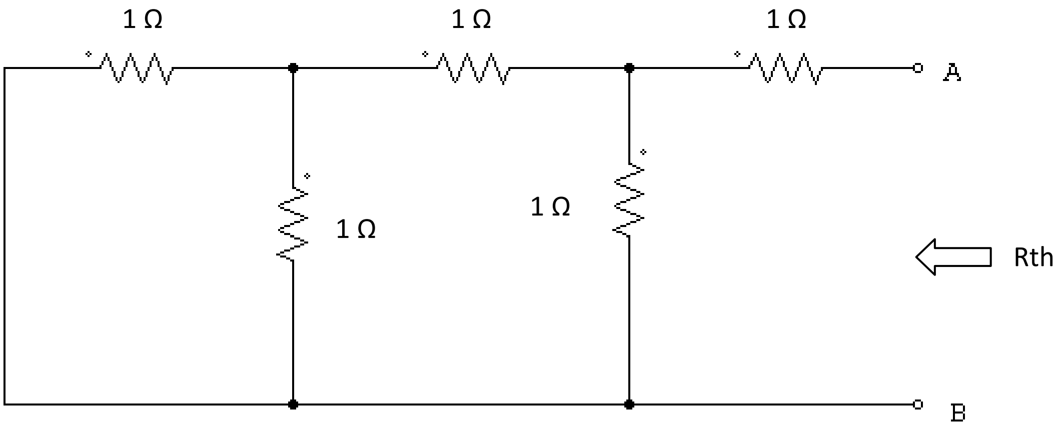
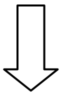


Fig.1.58



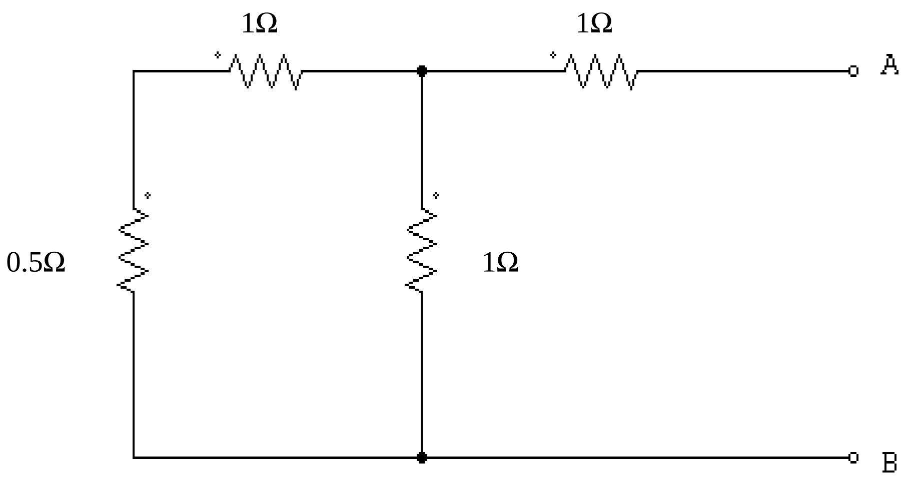


Fig.1.59

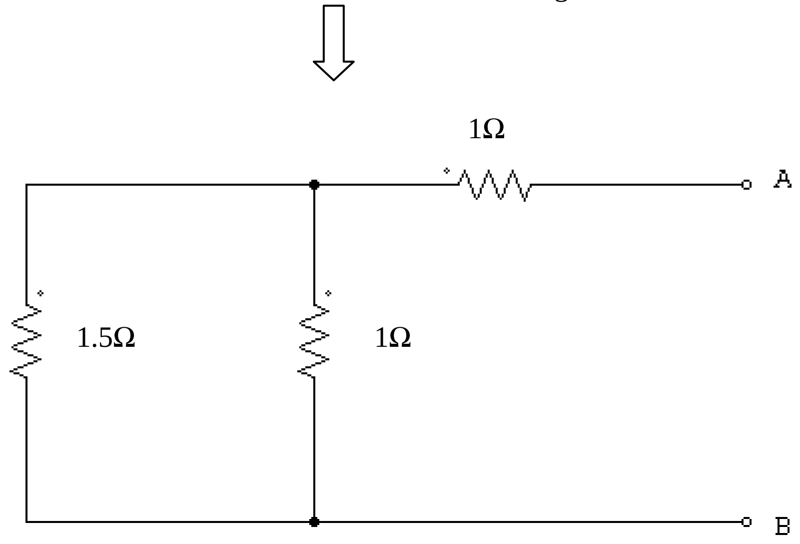


Fig.1.60

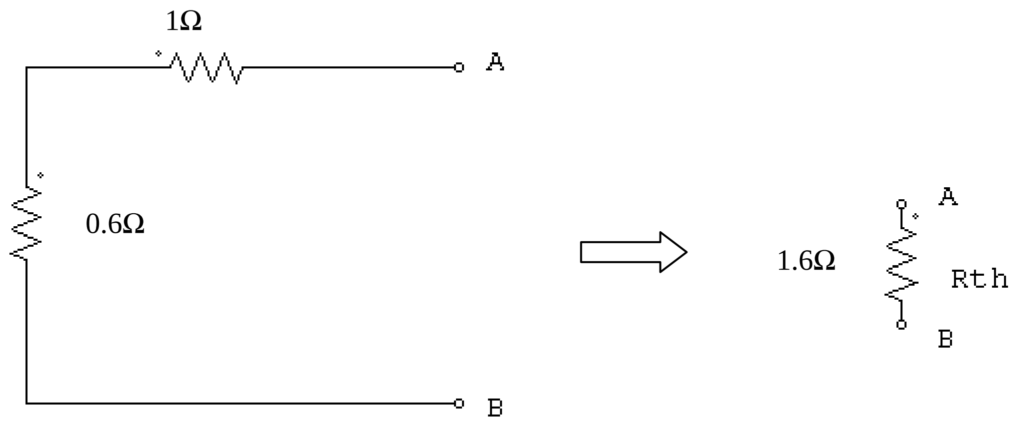


Fig.1.61

Step 2: To find V_{th} open the load resistance and find the voltage across AB and the circuit is redrawn.

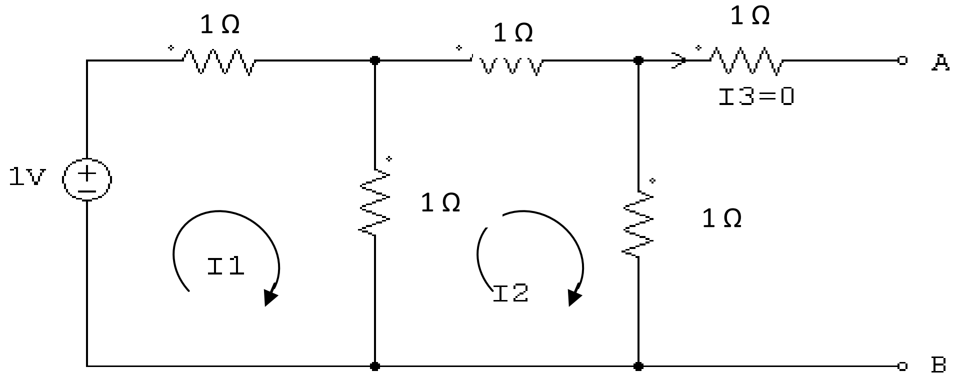


Fig.1.62

$$\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}2 & -1\\-1 & 3\end{bmatrix}\begin{bmatrix}I1\\I2\end{bmatrix}$$

$$I_2 = \frac{\begin{bmatrix}2 & 1\\-1 & 0\end{bmatrix}}{\begin{bmatrix}2 & -1\\-1 & 3\end{bmatrix}} = 0.2\ \text{A}$$

open circuit voltage across A,B

$V_{th} = 0.2\text{A} \times 1\,\Omega$

$= 0.2$ V

Step 3: Thevenin's equivalent circuit is drawn to find current

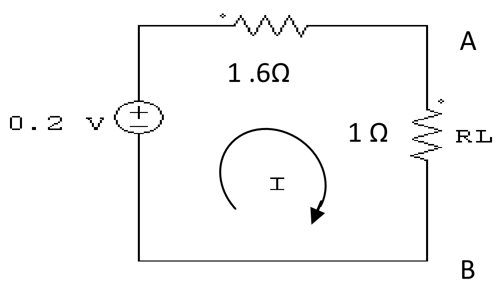


Fig.1.63

$$I = \frac{Vth}{Rth+RL} = \frac{0.2V}{1.6+1\Omega} = 0.0769\ \text{A}$$

$\boxed{I = 0.076\text{A}}$

Example Problem:

Find the current using Thevenin's Theorem flowing through the load resistance 1Ω in the given circuit in Fig.1.64

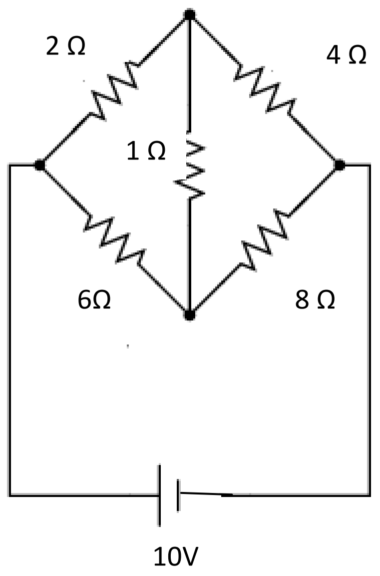


Fig.1.64

Step 1: To find Thevenin's resistance R_{th} open the load resistance & voltage source is replaced by its internal resistance and the circuit is redrawn as follows.

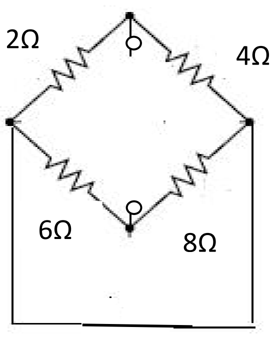


Fig.1.65

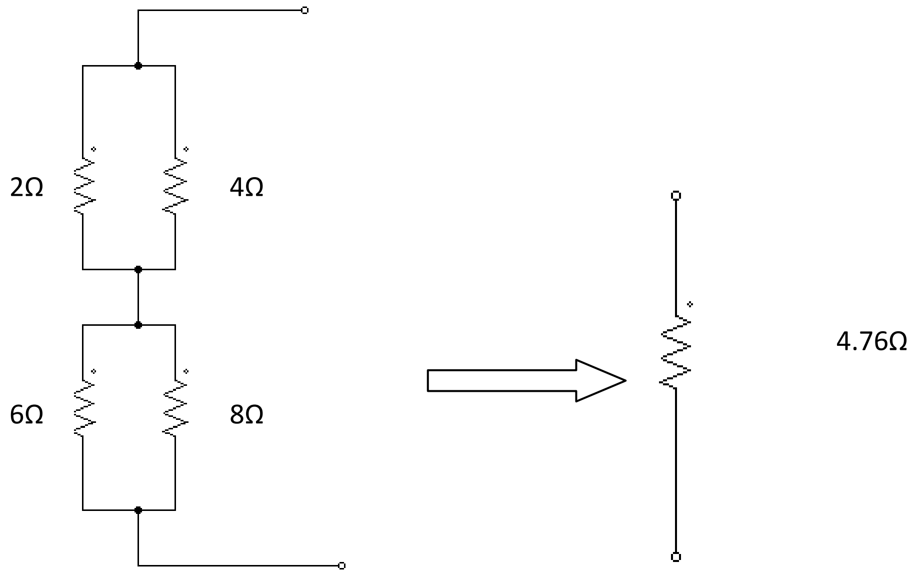


Fig.1.66

$R_{th} = 4.76\Omega$

Step 2: To find V_{th} open the load resistance and find the voltage across AB and the circuit is redrawn

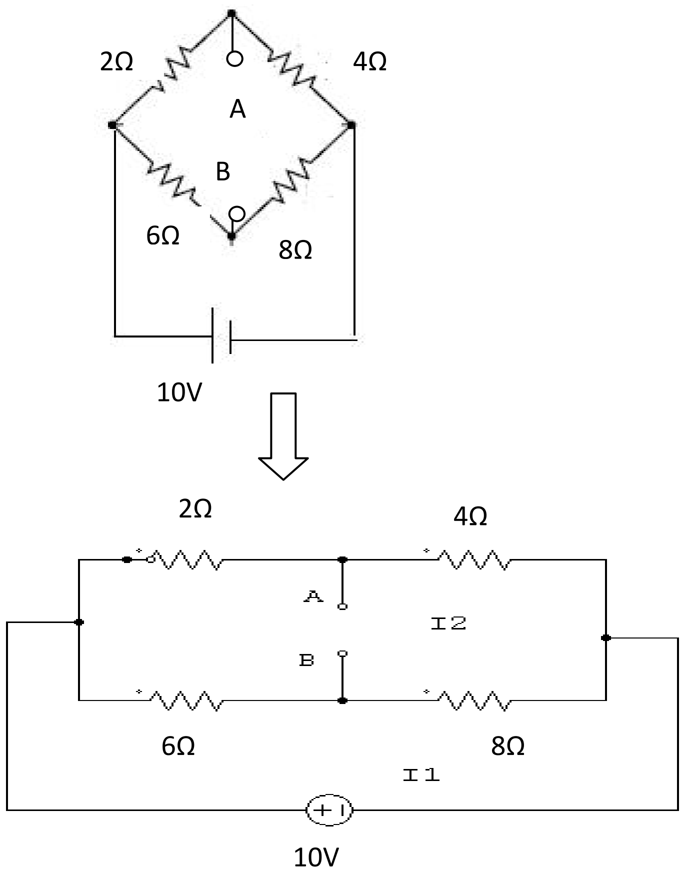


Fig.1.67

$I_1 = \frac{10V}{14\Omega} = 0.714A$

$I_2 = \frac{10V}{6\Omega} = 1.66A$

Potential at A = 1.66A x 4 = 6.64V

Potential at B = 0.714A x 8 = 5.712V

Potential difference between A & B is

=6.64 – 5.712 V

$\boxed{V_{th} = 0.93\ V}$

Step 3: Thevenin's equivalent circuit is drawn to find current flowing throguh load resistance R_L

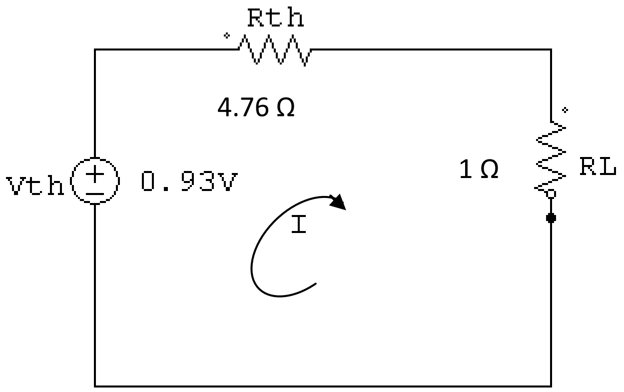


Fig.1.68

$I = \frac{Vth}{Rth+RL} = \frac{0.93}{4.76+1} = 0.162A$

$\boxed{I = 0.162A}$

1.8 NORTON’S THEOREM

Statement :

Any circuit with voltage source, resistances can be replaced by a single current source in parallel with single resistance, where the value of current source is equal to the current passing through the short circuit output terminals and the value of resistance is equal to the resistance seen into the output terminals, as shown in Fig.1.69.

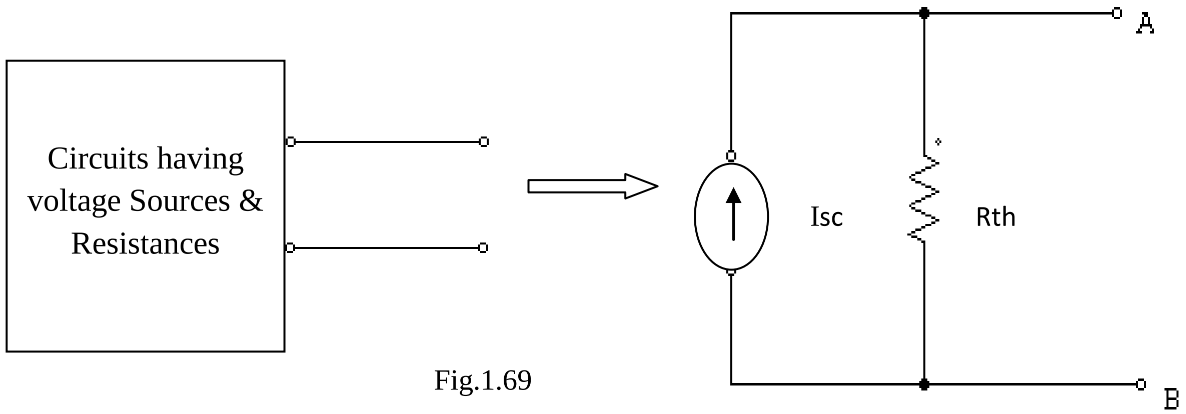


Fig.1.69

Isc – Short circuit current flowing through output terminals.

R_{Norton} – Total equivalent resistance seen into the network

Example:

Consider the circuit shown in figure 1.70

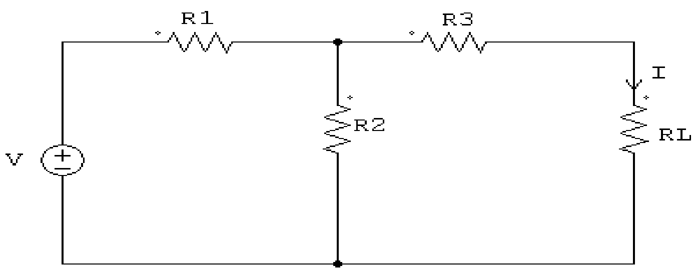


Fig.1.70

Let us find the current through R_L. By applying NORTON’S Theorem, the given circuit can be replaced by the NORTON’S equivalent circuit shown in Fig.1.71

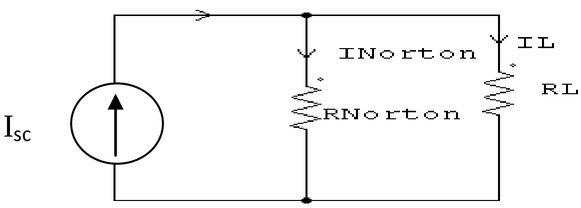


Fig.1.71

$I_{sc} = I_{Norton} + I_L$

$I_L = \frac{RNorton}{RNorton+RL}$ X I_{sc}

Method :

Step 1: Find Norton's equivalent resistance R_{Norton} for the given circuit. To find R_{Norton} , open the load resistance and replace the voltage source by its internal resistance (or) replace by short if it's internal resistance is zero. Circuit is redrawn as follows to find R_{Norton}.

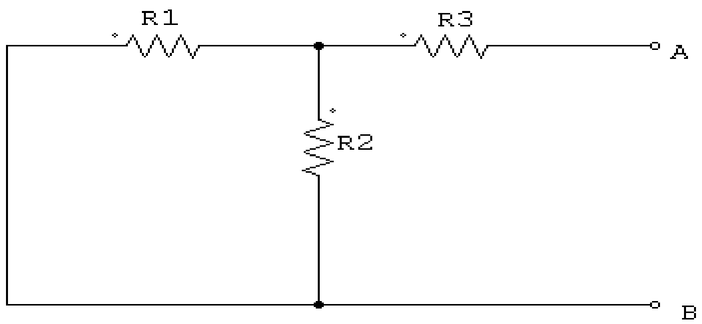


Fig.1.72

$R_N = R_1 || R_2 + R_3$

$= \frac{R1R2}{R1+R2} + R_3$

$$\boxed{R_N = \frac{R1R2+R2R3+R1R3}{R1+R2}}$$

Step 2: Find short circuit current I_{sc}. To find I_{sc}, short the load resistance and find the current flowing through the output terminals. Circuit is redrawn a follows.

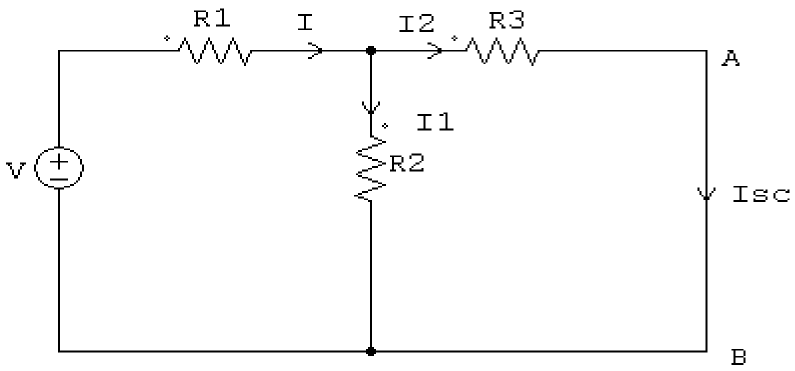


Fig.1.73

$I_{sc} = I_2 = \frac{R2}{R2+R3}$ x I

Step 3 : Draw the Norton's equivalent and find the current flowing through load resistance.

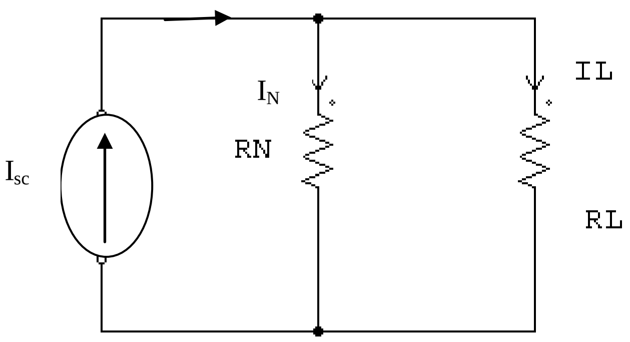


Fig.1.74

Current flowing through load resistance

$$I_L = \frac{RN}{RN+RL} \times I_{sc}$$

Example Problem :

Find the current flowing through load resistance in Fig.1.75 using Norton's theorem.

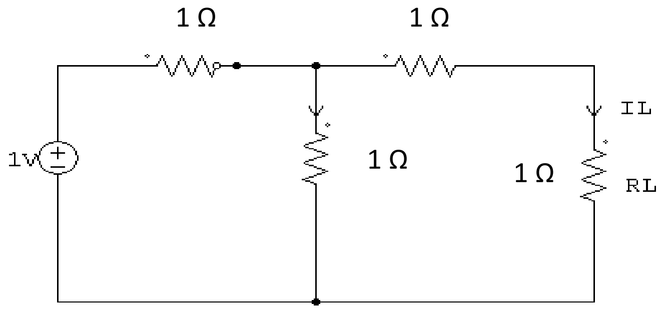


Fig.1.75

Step 1: To find Norton's resistance R_N, open the load resistance & voltage source is replaced by short and circuit is redrawn as follows.

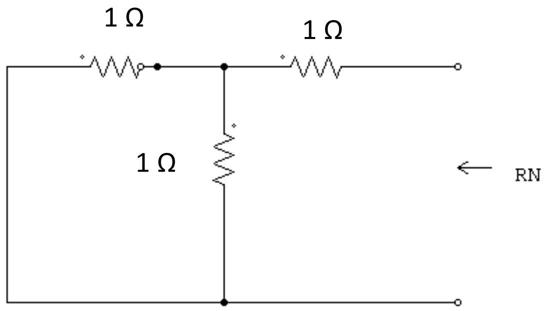


Fig.1.76

$R_N = 1.5\ \Omega$

Step 2: To find the short circuit current I_{sc} short the load resistance and find the current flowing through the output terminals.

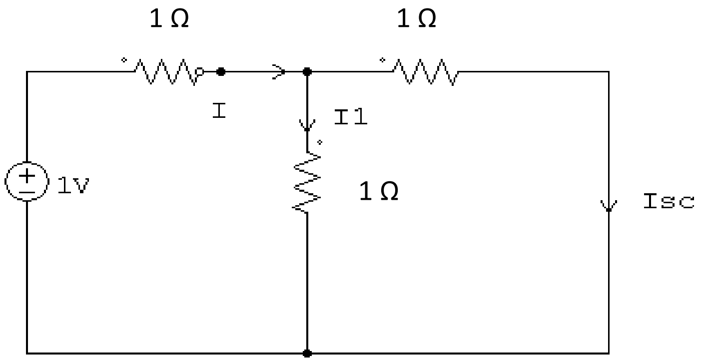


Fig.1.77

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} I1 \\ Isc \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$I_{sc} = \frac{\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}}{\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}}$$

I_{sc} = 1/3 = 0.33A

Step 3: Norton's equivalent circuit is drawn to find the load current.

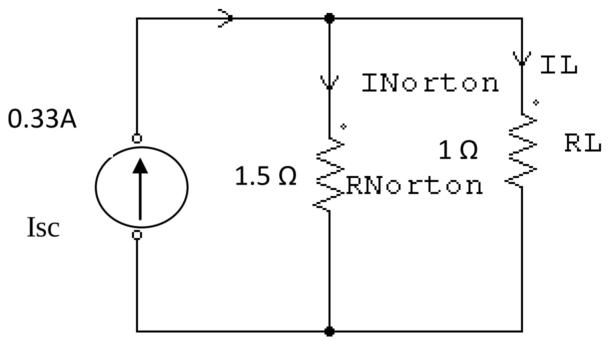


Fig.1.78

$$I_L = \frac{1.5}{1+1.5} \text{ x } 0.33$$

$$\boxed{I_L = 0.2A}$$

Example Problem :

Find the current I flowing through load resistance using Norton's theorem

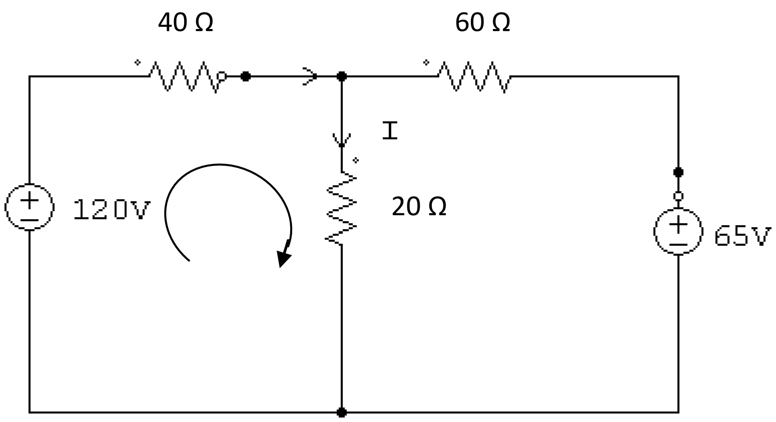


Fig.1.79

Step 1: To find Norton's resistance R_N open the load resistance & voltage source is replaced by short and circuit is redrawn as follows in Fig1.80.

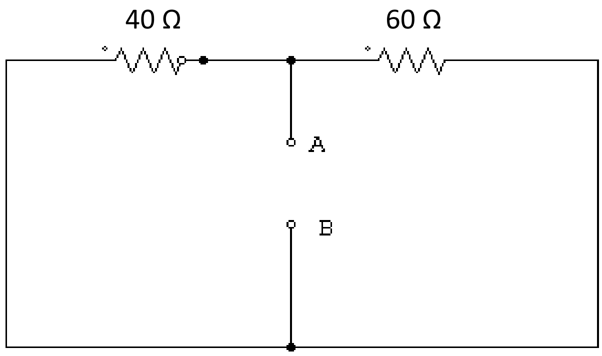


Fig.1.80

$$R_N = \frac{60 \, X \, 40}{60+40} = 24 \, \Omega$$

Step 2: To find the short circuit current I_{sc}, short the load resistance and find the current flows through the output terminals.

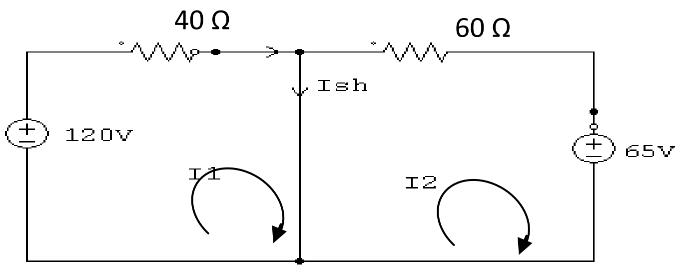


Fig.1.81

$$\begin{bmatrix}120\\-65\end{bmatrix} = \begin{bmatrix}I1\\I2\end{bmatrix} \begin{bmatrix}40 & 0\\0 & 60\end{bmatrix}$$

$$I_1 = \frac{\begin{bmatrix}120 & 0\\-65 & 60\end{bmatrix}}{\begin{bmatrix}40 & -0\\0 & 60\end{bmatrix}} = \frac{7200}{2400} = 3A$$

$$I_2 = \frac{\begin{bmatrix}40 & 120\\0 & -65\end{bmatrix}}{\begin{bmatrix}40 & 0\\0 & 60\end{bmatrix}} = \frac{-2600}{2400} = -1.083A$$

$I_{sh} = I_1 - I_2 = 3-(-1.083) = 4.083A$

Step 3: Norton's equivalent circuit is drawn to find the load current

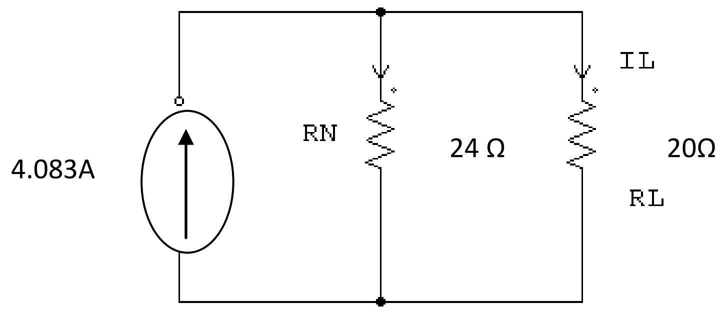


Fig.1.82

$I_L = \frac{RN}{RL+RN} \cdot I_{sh}$

$= \frac{24}{44}$ X 4.083

$\boxed{I_L = 2.22A}$

Example Problem:

Find the current I flowing through the load resistance R_L using Norton's theorem for the circuit shown in Fig.1.83

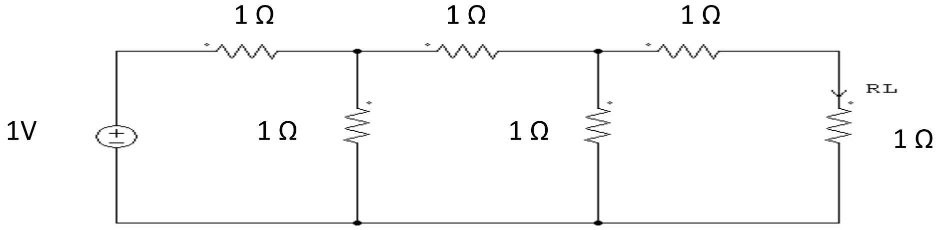


Fig.1.83

Step 1: To find Norton's resistance R_N, open the load resistance & voltage source is preplaced by short and circuit is redrawn as follows in Fig.1.84.

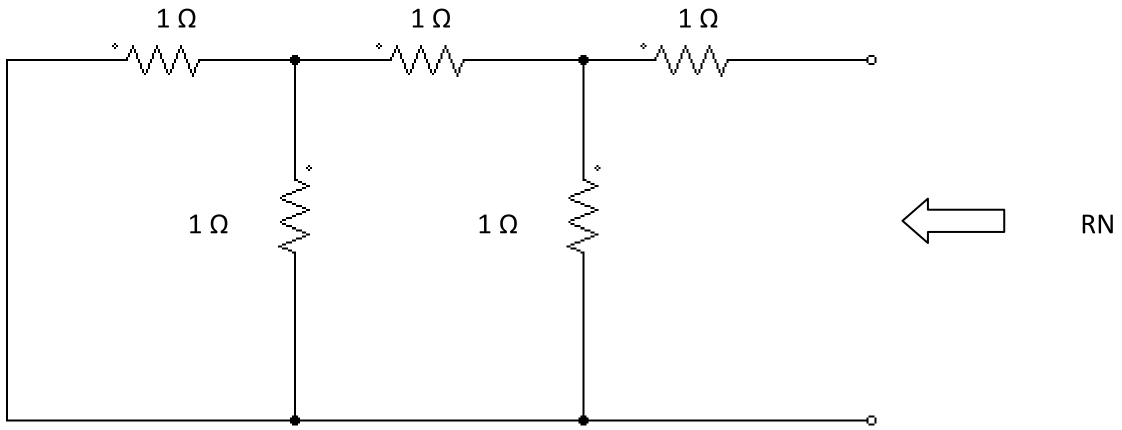


Fig.1.84

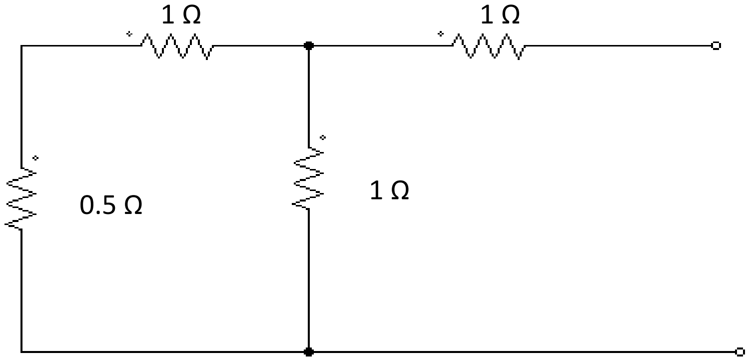


Fig.1.85

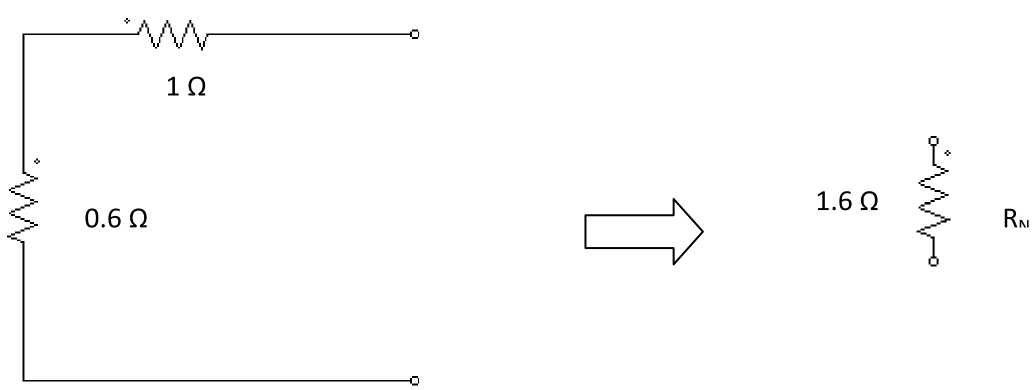


Fig.1.86

Step 2: To find the short circuit current I_{sc}, short the load resistance and find the current flows through the output terminals.

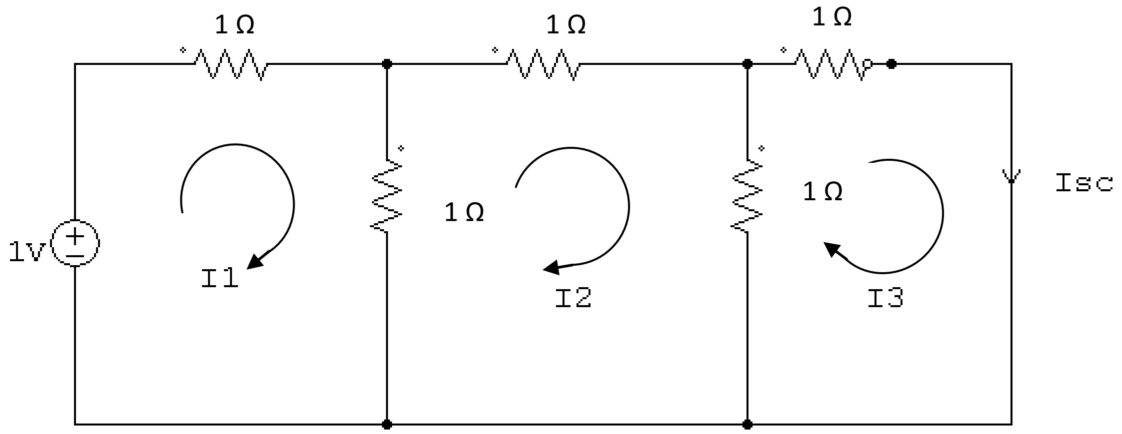


Fig.1.87

$$\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}2 & -1 & 0\\-1 & 3 & -1\\0 & -1 & 2\end{bmatrix}\begin{bmatrix}I1\\I2\\I3\end{bmatrix}$$

$$I_3 = \frac{\begin{bmatrix}2 & -1 & 0\\-1 & 3 & 0\\0 & -1 & 0\end{bmatrix}}{\begin{bmatrix}2 & -1 & 0\\-1 & 3 & -1\\0 & -1 & 2\end{bmatrix}}$$

$$= \frac{2(0-0)+1(0-0)+1(1-0)}{2(6-1)+1(-2+1)+0}$$

$I_{sc} = I_3 = \frac{1}{9} = 0.11A$

Step 3: Norton's equivalent circuit is drawn to find the load current

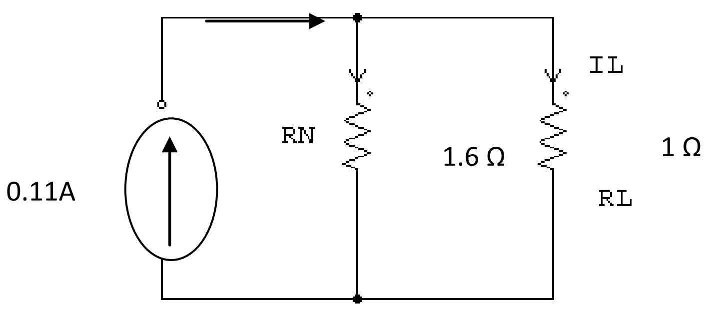


Fig.1.88

$I_L = \frac{RN}{RN+RL}$ X I_{sc}

$= \frac{1.6}{2.6}$ X 0.11

= 0.07 A

1.9 SUPER POSITION THEOREM

Statement

Any circuit containing two (or) more sources, the current in any resistor is equal to the algebraic sum of the separate currents in the resistor when each source acts separately. While one source is applied, the other voltage and current sources are replaced by short circuit and open circuit across their terminals.

This theorem is valid only for linear circuit.

Example :

Consider the circuit shown in figure 1.89

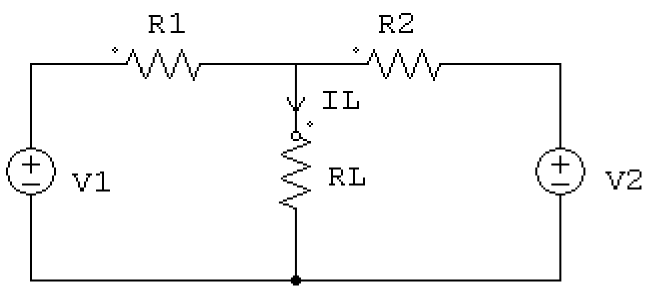
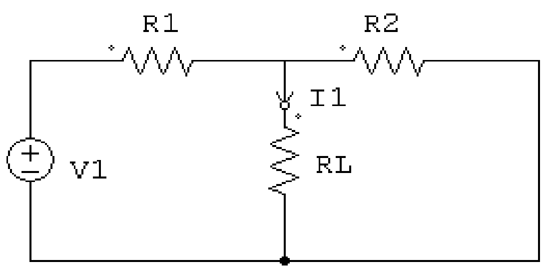


Fig.1.89

Let us find the current through R_L. By applying super position theorem, the current I_L flowing through load resistance can be found as given below.



+

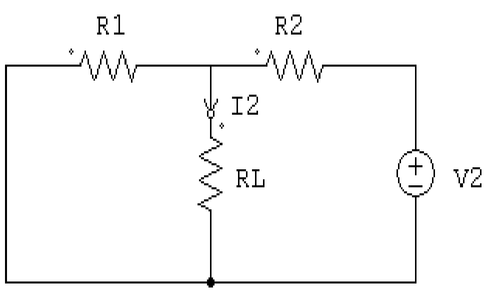


Fig.1.90 Fig.1.91

$$\boxed{I_L = I_1 \pm I_2}$$

When V_1 & V_2 is in such a way that I_1 and I_2 flows in the same direction, they are added together, otherwise the difference between I_1 & I_2 is the resultant current.

Method :

Step 1: To find the current I_1 through the load resistance R_L, due to the source V_1, the other source V_2 is replaced by short (internal resistance) and circuit is redrawn as follows.

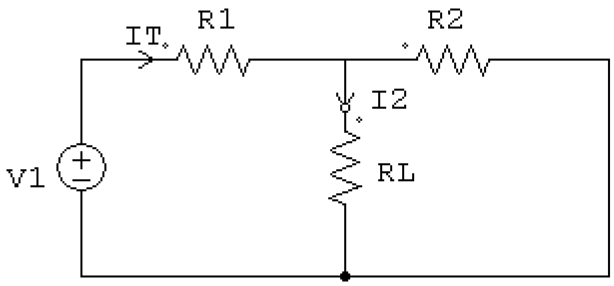


Fig.1.92

$I_T = \frac{V1}{R1+\frac{R2RL}{R2+RL}}$

$I_T = \frac{V1(R2+RL)}{R1(R2+RL)+R2RL}$

$I_1 = \frac{R2}{R2+RL}$ X I_T

Step 2: To find the current I_2 through the load resistance R_L, due to the source V_2, the other source V_1 is replaced by short and circuit is redrawn as follows.

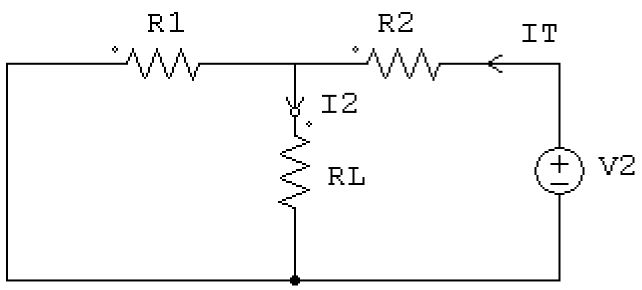


Fig.1.93

$I_T = \frac{V2}{R2+\frac{R1RL}{R1+RL}}$

$I_2 = \frac{R1}{R1+RL}$ X I_T

Step 3: The algebraic sum (or) difference of I_1 & I_2 is the load current I_L based on the direction of I_1 & I_2.

If I_1 & I_2 flows in same direction.

$I_L = I_1 + I_2$

other wise $I_L = I_1 - I_2$

Example problem :

Find the current flowing through load resistance using super position theorem.

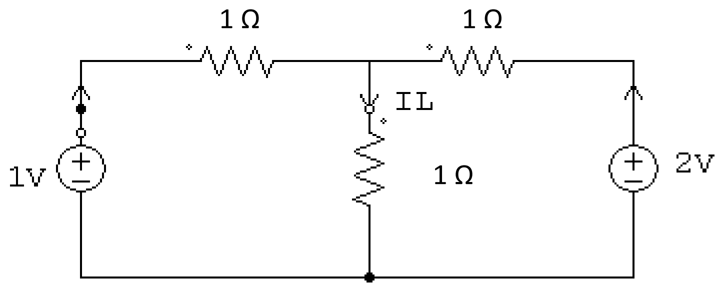


Fig.1.94

Step 1*:* To find the current I_1 through the load resistance, due to 1V source, 2V source is replaced by short and the circuit is redrawn as follows.

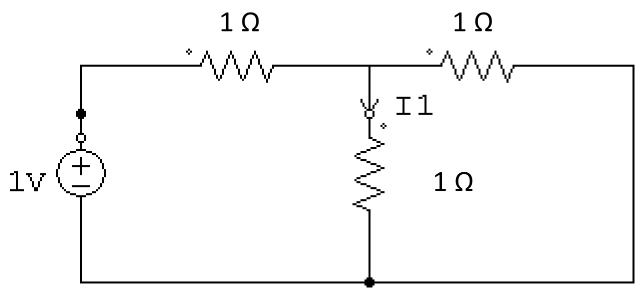


Fig.1.95

$R_{eq} = 1 + \frac{1}{2} = 1.5\ \Omega$; $I_T = \frac{1}{1.5} = 0.66A$

$I_1 = \frac{1}{2}$ X $0.66 = 0.33A$

Step 2: To find the current I_2 through load resistance, due to 2V source, 1V source is replaced by short and the circuit is redrawn as follows.

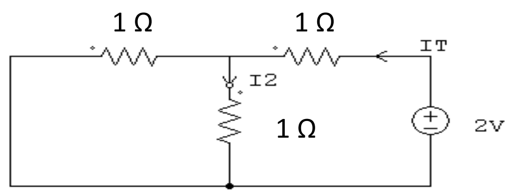


Fig.1.96

$R_{eq} = 1 + \frac{1}{2} = 1.5\ \Omega$; $I_T = \frac{2}{1.5} = 1.33A$

$I_2 = \frac{1}{2}$ X $1.33 = 0.665A$

Step 3: Since, I_1 and I_2 flows in the same direction,

$I_L = I_1 + I_2$

$= 0.33 + 0.665 = 0.995A$

$\boxed{I_L = 0.995A}$

Example Problem:

Find the current flowing through load resistance using super position theorem.

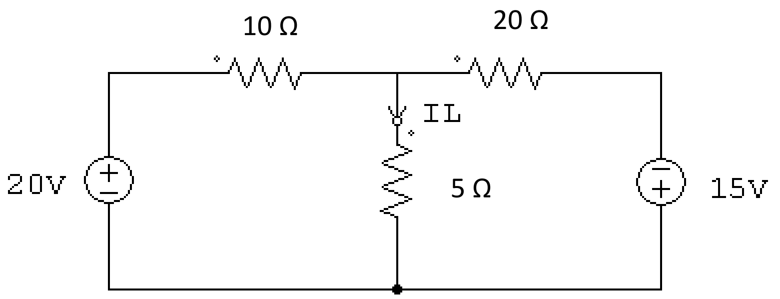


Fig.1.97

Step 1:

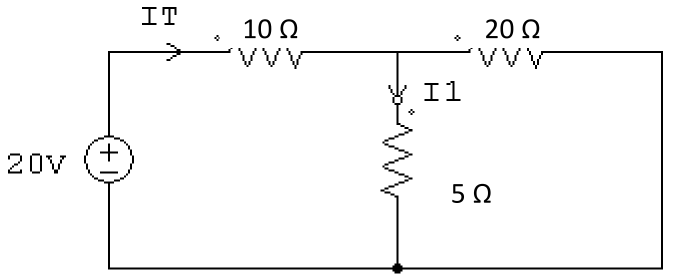


Fig.1.98

$R_{eq} = 10 + \frac{20 \times 5}{20+5}$; $I_T = \frac{20}{14} = 1.428A$

$R_{eq} = 14\ \Omega$

$I_1 = \frac{20}{25}$ X $1.428 = 1.142A$ $\boxed{I_1 = 1.142A}$

Step 2:

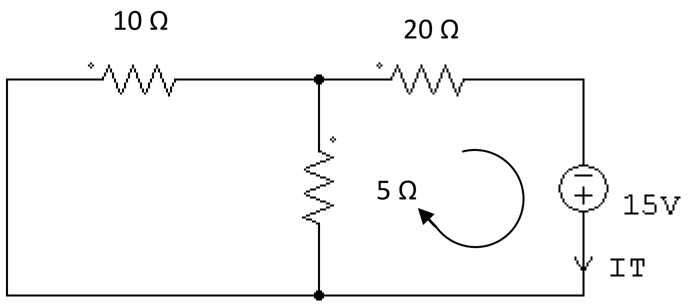


Fig.1.99

R_{eq} =20 + $\frac{10 \times 5}{10+5}$ = 23.3Ω ; I_T = $\frac{15}{23.3}$ = 0.644A

I_2 = $\frac{10}{15}$ X 0.644 = 0.429A

$\boxed{I_2 = 0.429A}$

Step 3: Since the current I_1 & I_2 flows in the opposite direction,
$I_L = I_1 - I_2$

= 1.142 – 0.429 = 0.713 A

$\boxed{I_L = 0.713A}$

Example Problem:

Find the current flowing through 4Ω resistor using super position theorem

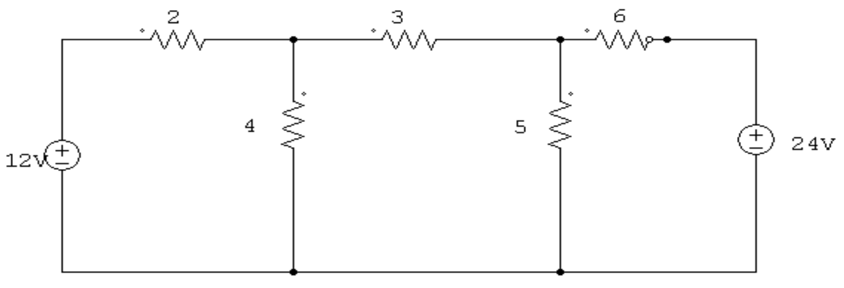


Fig.1.100

Step 1: considering 12V source, while other source is shorted

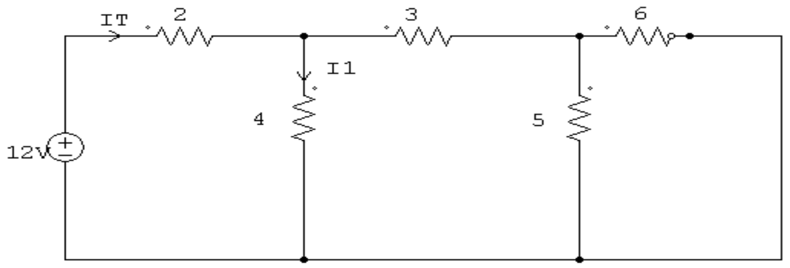


Fig.1.101

$I_T = 2.75A$

$I_1 = \frac{5.72}{9.72} X\ 2.75$

$\boxed{I_1 = 1.618A}$

Step 2: considering 24V source, while other source is shorted.

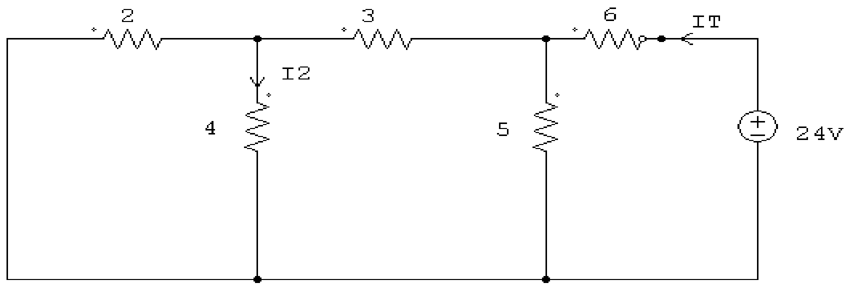


Fig.1.102

$I_T = 2.88A$

$I_2 = \frac{2}{6} X\ 1.543$

$\boxed{I_2 = 0.514A}$

Step 3 : Since I_1 & I_2 flows in the same direction through 4 resistor.

$I_L = I_1 + I_2$

$\boxed{I_L = 2.132A}$

1.10 MAXIMUM POWER TRANSFER THEOREM

In any electric circuit, the electrical energy from the supply is delivered to the load where it is converted into a useful work. Practically, the entire supplied power will not present at load due to the heating effect and other constraints in the network. Therefore, there exist a certain difference between drawing and delivering powers.

The load size always affects the amount of power transferred from the supply source, i.e., any change in the load resistance results to change in power transfer to the load. Thus, the maximum power transfer theorem ensures the condition to transfer the maximum power to the load.

Maximum Power Transfer Theorem Statement

The maximum power transfer theorem states that in a linear, bilateral DC network, maximum power is delivered to the load when the load resistance is equal to the internal resistance of a source.

If it is an independent voltage source, then its series resistance (internal resistance Rs) or if it is independent current source, then its parallel resistance (internal resistance Rs) must be equal to the load resistance RL to deliver maximum power to the load.

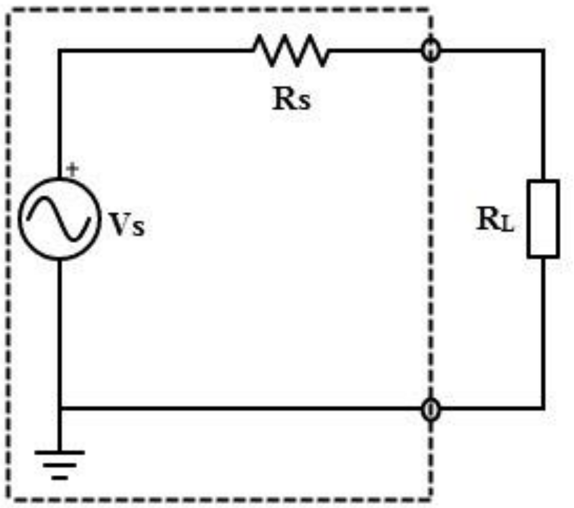


Fig.1.103

Proof of Maximum Power Transfer Theorem

The maximum power transfer theorem ensures the value of the load resistance, at which the maximum power is transferred to the load.

Consider the below DC two terminal network (left side circuit), to which the condition for maximum power is determined, by obtaining the expression of power absorbed by load with use of mesh or nodal current methods and then deriving the resulting expression with respect to load resistance R_L.

But this is quite a complex procedure. But in previous articles we have seen that the complex part of the network can be replaced with the Thevenin's equivalent as shown below.

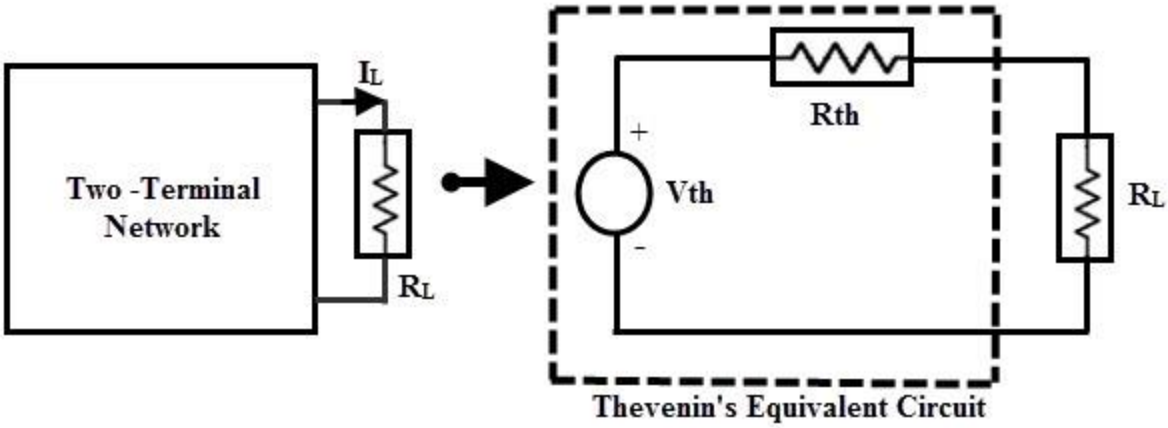


Fig.1.104

The original two terminal circuit is replaced with the Thevenin's equivalent circuit across the variable load resistance. The current through the load for any value of load resistance is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

The power absorbed by the load is

$$P_L = I_L{}^2 \times R_L$$

$$= \left[\frac{V_{Th}}{R_{Th} + R_L}\right]^2 \times R_L \qquad \text{……………(1)}$$

Form the above expression the power delivered depends on the values of R_{TH} and R_L. However the Thevenin's equivalent is constant, the power delivered from this equivalent source to the load entirely depends on the load resistance R_L. To find the exact value of R_L, we apply differentiation to P_L with respect to R_L and equating it to zero as

$$\frac{dP(R_L)}{dR_L} = V_{Th}^{\ 2}\left[\frac{(R_{Th}+R_L)^2 - 2R_L \times (R_{Th}+R_L)}{\left(R_{Th}+R_L\right)^4}\right] = 0$$

$$\Rightarrow (R_{Th}+R_L) - 2R_L = 0$$

$$\Rightarrow R_L = R_{Th}$$

Therefore, this is the condition of matching the load where the maximum power transfer occurs when the load resistance is equal to the Thevenin's resistance of the circuit. By substituting $R_{th} = R_L$ in equation 1 we get

The maximum power delivered to the load is,

$$P_{\max} = \left[\frac{V_{Th}}{R_{Th}+R_L}\right]^2 \times R_L \Bigg|_{R_L = R_{Th}}$$

$$= \frac{V_{Th}^{\ 2}}{4R_{Th}}$$

Total power transferred from source is

$P_T = I_L^2 (R_{TH} + R_L)$

$= 2 I_L^2 R_L$ …………….(2)

Hence , the maximum power transfer theorem expresses the state at which maximum power is delivered to the load , that is , when the load resistance is equal to the Thevenin's equivalent resistance of the circuit. Fig.1.105 shows a curve of power delivered to the load with respect to the load resistance. Note that the power delivered is zero when the load resistance is zero as there is no voltage drop across the load during this condition. Also, the power will be maximum, when the load resistance is equal to the internal resistance of the circuit (or Thevenin's equivalent resistance). Again, the power is zero as the load resistance reaches to infinity as there is no current flow through the load.

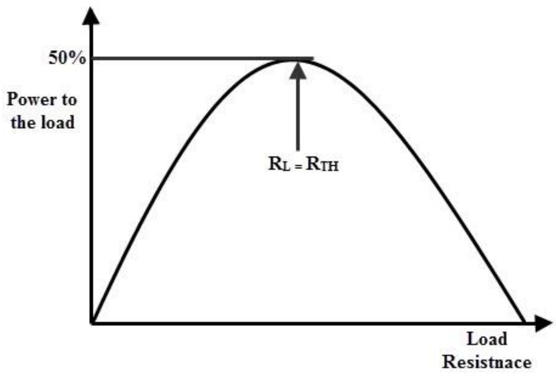


Fig.1.105

Power Transfer Efficiency

We must remember that this theorem results maximum power transfer but not a maximum efficiency. If the load resistance is smaller than source resistance, power dissipated at the load is reduced while most of the power is dissipated at the source then the efficiency becomes lower.

Consider the total power delivered from source equation (equation 2), in which the power is dissipated in the Thevenin's equivalent resistance R_{TH} by the voltage source V_{TH}.

Therefore, the efficiency under the condition of maximum power transfer is

Efficiency = Output / Input × 100

$= I_L^2 R_L / 2 I_L^2 R_L \times 100$

= 50 %

Hence, at the condition of maximum power transfer, the efficiency is 50%, that means a half percentage of generated power is delivered to the load and at other conditions small percentage of power is delivered to the load , as indicated in efficiency verses maximum power transfer curves below.

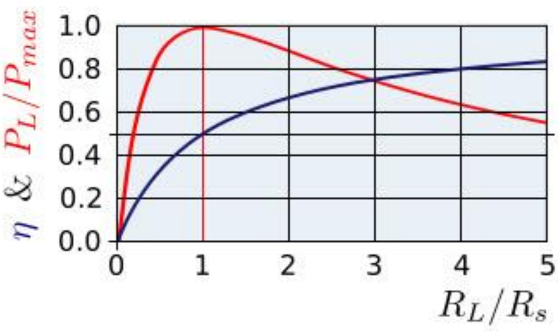


Fig. 1.106

For some applications, it is desirable to transfer maximum power to the load than achieving high efficiency such as in amplifiers and communication circuits.

On the other hand, it is desirable to achieve higher efficiency than maximised power transfer in case of power transmission systems where a large load resistance (much larger value than internal source resistance) is placed across the load. Even though the efficiency is high the power delivered will be less in those cases.

Maximum Power Transfer Theorem for AC Circuits

It can be stated as in an active network, the maximum power is transferred to the load when the load impedance is equal to the complex conjugate of an equivalent impedance of a given network as viewed from the load terminals.

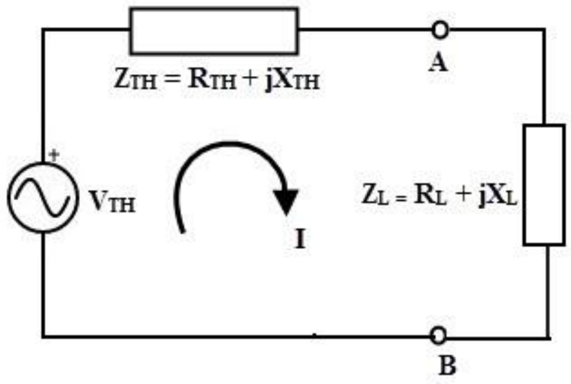


Fig. 1.107

Consider the above Thevenin's equivalent circuit across the load terminals in which the current flowing through the circuit is given as

$I = V_{TH} /(Z_{TH} + Z_L)$

Where $Z_L = R_L + jX_L$

$Z_{TH} = R_{TH} + jX_{TH}$

Therefore, $I = V_{TH} / (R_L + jX_L + R_{TH} + jX_{TH})$

$= V_{TH} / ((R_L+ R_{TH}) + j(X_L + X_{TH}))$

The power delivered to the load,

$P_L = I^2 R_L$

$P_L = V^2_{TH} \times R_L / ((R_L+ R_{TH})^2 + (X_L + X_{TH})^2)$ ……(1)

For maximum power the derivative of the above equation must be zero, after simplification we get

$X_L + X_{TH} = 0$

$X_L = – X_{TH}$

Putting the above relation in equation 1, we get

$P_L = V^2_{TH} \times R_L / ((R_L + R_{TH})^2$

Again for maximum power transfer, derivation of above equation must be equal to zero, after simplification we get

$R_L + R_{TH} = 2 R_L$

$R_L = R_{TH}$

Hence, the maximum power will transferred to the load from source, if $R_L = R_{TH}$ and $X_L = – X_{TH}$ in an AC circuit. This means that the load impedance should be equal to the complex conjugate of equivalent impedance of the circuit,

$Z_L = Z_{TH}$

Where Z_{TH} is the complex conjugate of the equivalent impedance of the circuit.

This maximum power transferred, $P_{max} = V^2_{TH} / 4 R_{TH}$ or $V^2_{TH} / 4 R_L$

Applying Maximum Power Transfer Example to DC circuit

Consider the below circuit to which we determine the value of the load resistance that receives the maximum power from the supply source and the maximum power under the maximum power transfer condition.

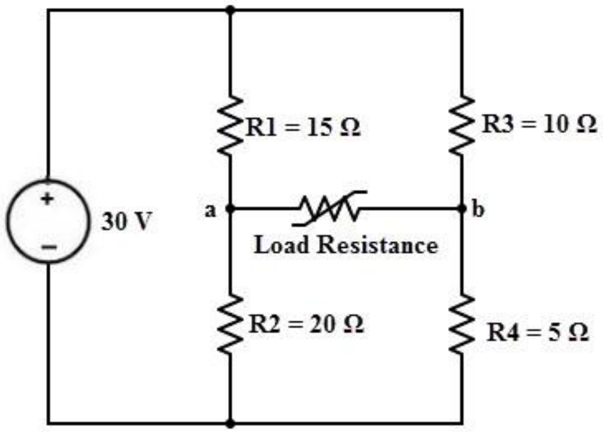


Fig. 1.108

1. Disconnect the load resistance from the load terminals a and b. To represent the given circuit as Thevenin's equivalent, we are to determine the Thevenin's voltage V_{TH} and Thevenin's equivalent resistance R_{TH}.

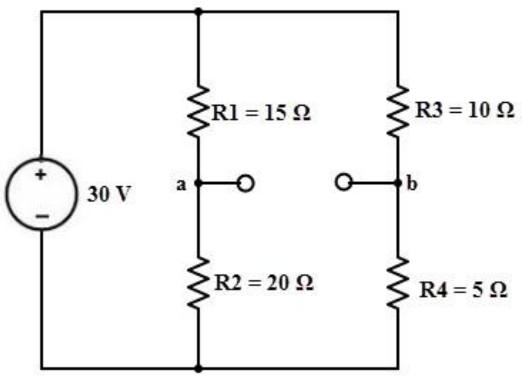


Fig. 1.109

The Thevenin's voltage or voltage across the terminals ab is $V_{ab} = V_a – V_b$

Va = V × R2 / (R1 + R2)

= 30 × 20 /×(20 + 15)

= 17.14 V

Vb = V × R4/ (R3 + R4)

= 30 × 5 /(10 + 5)

= 10 V

Vab = 17.14 – 10

= 7.14 V

V_{TH} = Vab = 7.14 Volts

2. Calculate the Thevenin's equivalent resistance R_{TH} by replacing sources with their internal resistances (here assume that voltage source has zero internal resistance so it becomes a short circuited).

Thevenin's equivalent resistance or resistance across the terminals ab is

R_{TH} = Rab = [R1R2 / (R1 + R2)] + [R3R4 /(R3 + R4)]

= [(15 × 20) / (15 + 20)] + [(10 × 5) / (10+ 5)]

= 8.57 + 3.33

R_{TH} = 11.90 Ohms

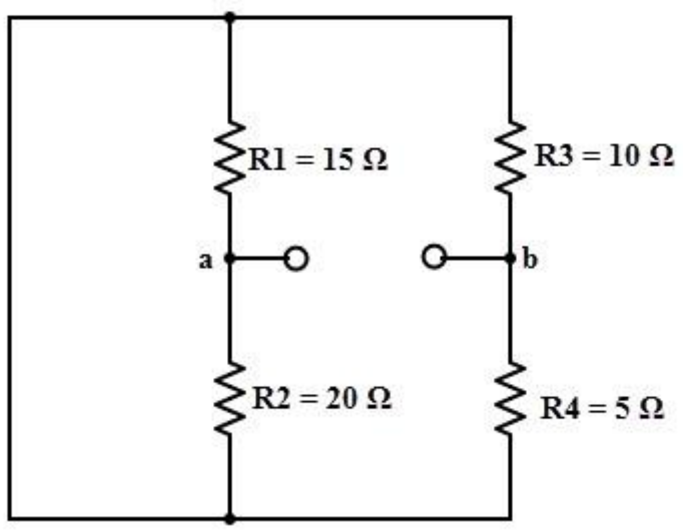


Fig. 1.110

The Thevenin's equivalent circuit with above calculated values by reconnecting the load resistance is shown below.

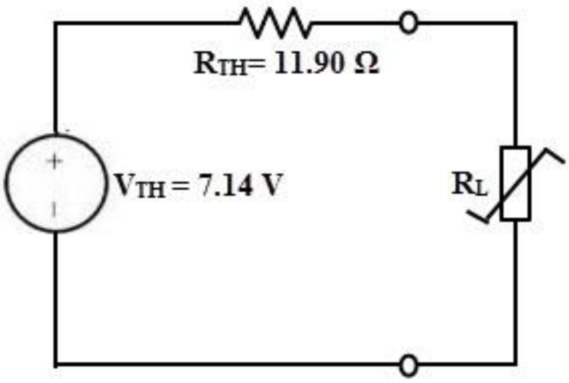


Fig. 1.111

From the maximum power transfer theorem, R_L value must equal to the R_{TH} to deliver the maximum power to the load.

Therefore, $R_L = R_{TH} = 11.90$ Ohms

And the maximum power transferred under this condition is,

$Pmax = V^2_{TH} / 4\ R_{TH}$

$= (7.14)^2 / (4 \times 11.90)$

$= 50.97 / 47.6$

$= 1.07$ Watts

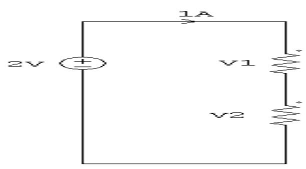
REVIEW QUESTIONS

2 MARK QUESTIONS:

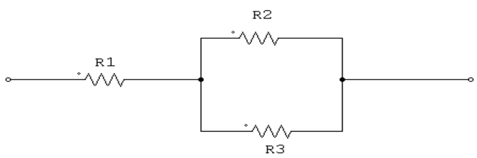
1. Define voltage
2. Define current
3. Define energy
4. Define power
5. What is the relation between energy and power?
6. What is resistance?
7. What is the power in watts if energy equal to 200 J is used in 1 sec?
8. What is a circuit?
9. State Ohm's Law
10. A 20Ω resistor is connected across a 20V Battery. How much current flows through the Resistor ?
11. Define inductance
12. Define capacitance
13. State Kirchoff's voltage law
14. State Kirchoff's current law
15. What is the equivalent resistance when two resistors R1 & R2 are connected in series?
16. What is the equivalent resistance when two resistors R1 & R2 are connected in parallel?
17. State Thevenin's Theorem
18. State Norton's Theorem
19. State Super position Therorem
20. State Maximum power Transfer Theroem

3 Mark Questions:

1. Current through a 10 Ω resistor is 2A. What is the power consumed by the resistor ?
2. Derive the energy stored in an inductor
3. Derive the energy stored in capacitor
4. Validate Kirchoff's voltage law for the given circuit



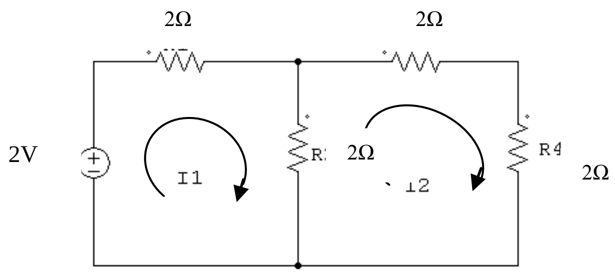
5. Find the equivalent resistance when R1=2Ω, R2=5Ω, R3=5Ω



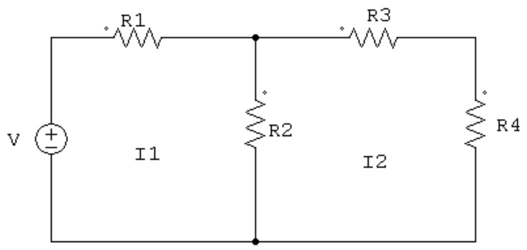
6. Draw the Thevenin's Equivalent circuit
7. Draw the Norton's Equivalent circuit

10 Mark Questions:

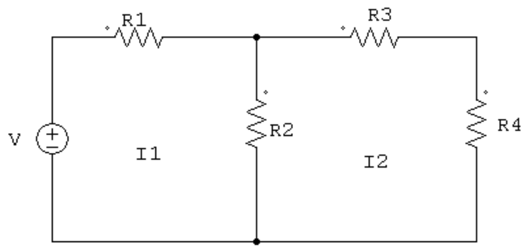
1. Three resistors 2Ω, 4Ω and 10Ω are connected in parallel across 12V battery. Find the current through each resistor. Also find the power dissipated in each resistor
2. A 10Ω resistor is connected in series with a parallel combination of three resistors 4Ω,10Ω and 20Ω respectively and is connected with a 50V DC power supply. Calculate the current through each resistor and total power dissipated.
3. A Wheatstone bridge ABCD is arranged as follows: AB=1Ω. BC=2Ω, CD=3Ω and DA=4Ω. A Galvanometer of resistance 10Ω is connected between B and D. A Battery of 4V is connected between A and C. Calculate the current in the Galvanometer by using Kirchoff's law.
4. A Wheatstone bridge ABCD is arranged as follows: AB=1Ω. BC=2Ω, CD=3Ω and DA=4Ω. A Galvanometer of resistance 10Ω is connected between B and D. A Battery of 4V is connected between A and C. Calculate the current in the Galvanometer by using Thevenins Theroem
5. Find the current flowing through each resistor using mesh current method for the circuit shown in the following figure.



6. Find the current flowing through each resistor using Thevenin's Theroem for the circuit shown in figure 1.37



7. Find the current flowing through each resistor using Norton's Theroem for the circuit shown in figure 1.37



UNIT – II- A C CIRCUITS

2.1 AC FUNDAMENTALS

2.1.1 AC WAVE FORMS

An alternating voltage and current wave forms are defined as voltage and current that fluctuate with time periodically with change in polarity & direction.

SINUSOIDAL WAVE AND NON-SINUSOIDAL WAVE

Alternating voltages and currents are represented by sinusoidal wave. The sinusoidal wave is generally referenced as sine wave.

Non-sinusoidal waveforms are waveforms that are not pure sine waves. Examples of non-sinusoidal waveforms include square waves, rectangular waves, triangle waves, spiked waves, trapezoidal waves and sawtooth waves.

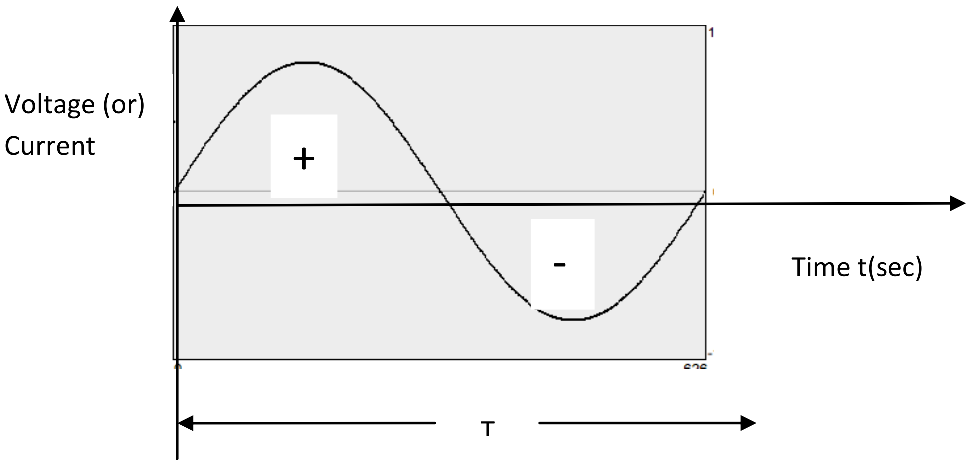


Fig.2.1

- The sinusoidal waveform changes its magnitude and direction with time. If we start at time t = 0, the wave goes to a maximum value and returns to zero, and then decreases to a negative maximum value before returning to zero as shown in Fig.2.1.
- During the positive portion of voltage, the current flows in one direction and during the negative portion of voltage, the current flows in opposite direction.
- The complete positive and negative portion of the sine wave is called one cycle of the sine wave.
- The time taken for one wave to complete one full cycle is called period T
- The frequency of a wave is defined as the number of cycles that a sine wave completes in one second. Frequency is measured in Hertz.

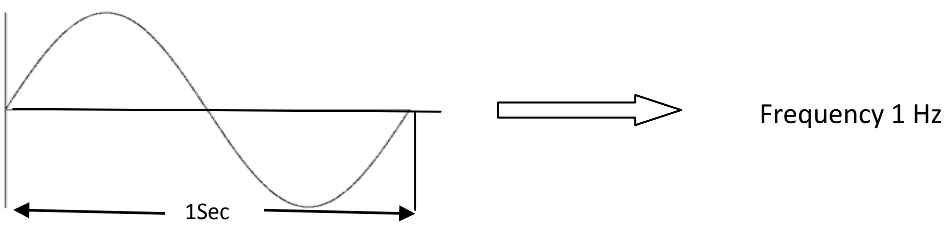


Fig.2.2

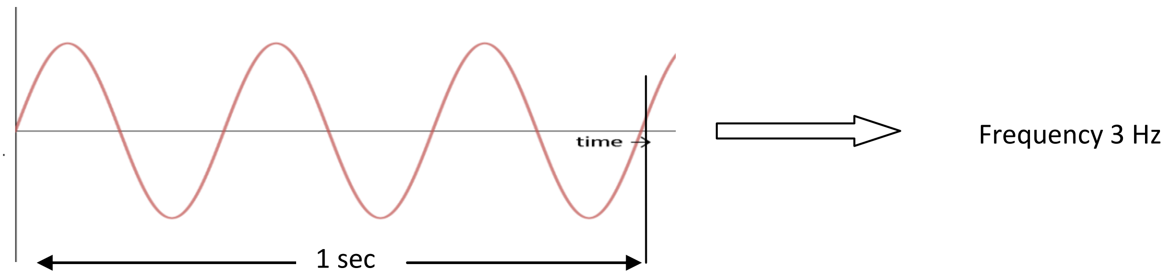


Fig.2.3

The relation between period and frequency is given by

$f = \frac{1}{T}$

Example Problem:

Find the period and frequency of given sine wave.

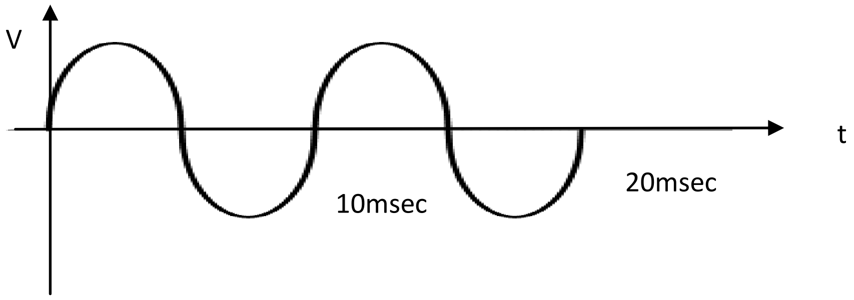


Fig.2.4

The sine wave takes 10m seconds to complete one period in each cycle.

T = 10msec.

Frequency $f = \frac{1}{T} = \frac{1}{10ms}$ = 100 Hz

2.1.2 ANGULAR RELATION OF A SINE WAVE

A sine wave can be expressed in terms of an angular measurement. This angular measurement is expressed in degrees (or) radians.

A radian is defined as the angular distance measured along the circumference of a circle which is equal to the radius of the circle. One radian is equal to 57.3^{o}. In a $360\,^{o}$ revolution, there are 2π radians.

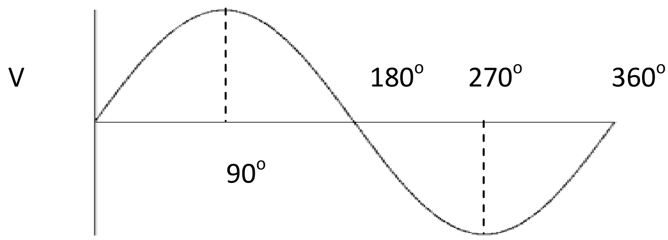


Fig.2.5

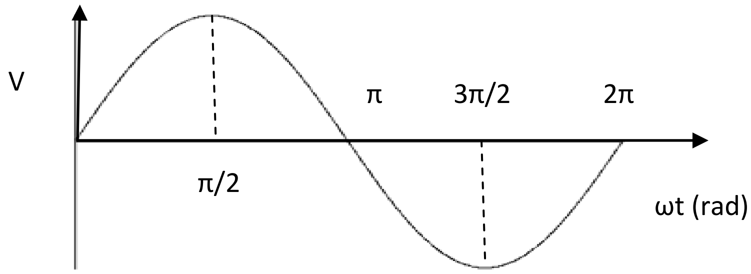


Fig.2.6

2.2 PHASE OF A SINE WAVE

The phase of a sine wave is an angular measurement that specifies the position of the sine wave relative to a reference. The wave shown in Fig. 2.7 is taken as the reference wave.

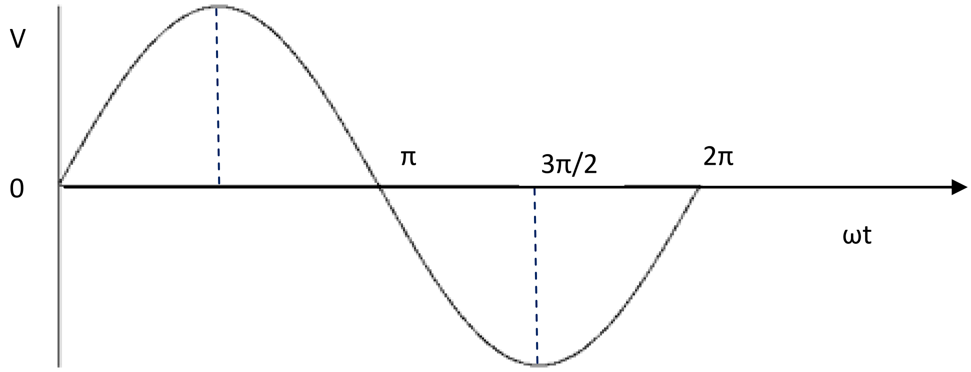


Fig.2.7

When the sine wave is shifted right (or) left with reference to the wave, as shown in Fig.2.8 (or) Fig.2.9, there occurs a phase shift.

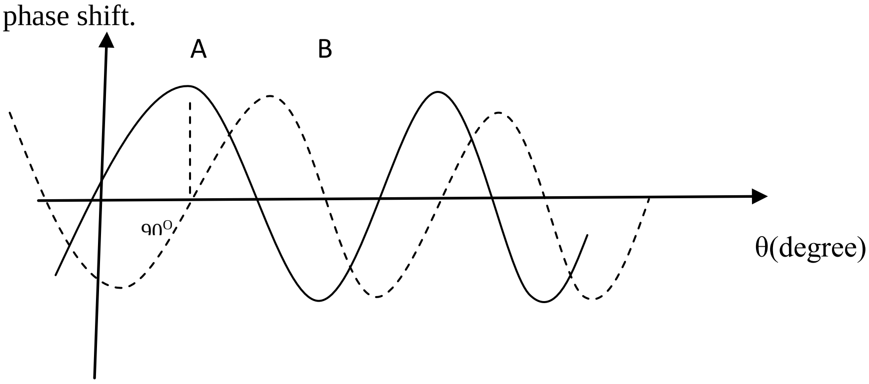


Fig.2.8

sine wave is shifted right by 90^O shown by dotted lines. B is lagging behind wave form A by 90^O

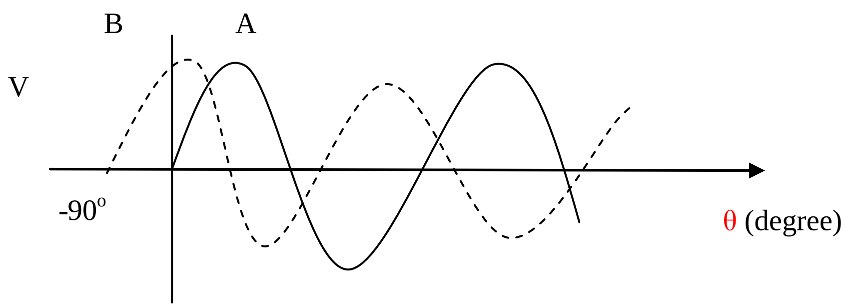


Fig.2.9

sine wave is shifted left by 90^O shown in dotted lines. B is leading wave form A by 90^O

Sine wave equation :

Let the sine wave of voltage is represented as shown in Fig.2.10

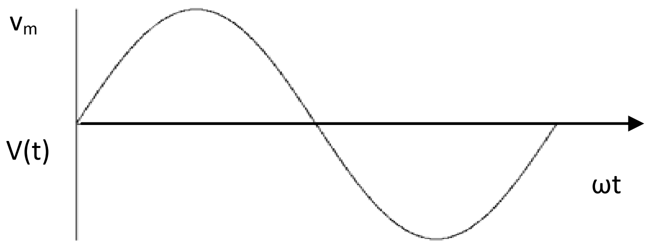


Fig.2.10

The amplitude of sine wave is V_m represented in vertical axis. The angular measurement is represented on horizontal axis. Sine wave of voltage is represented by the equation

$V(t) = V_m \sin\omega t$

$V(t)$ = Instantaneous value

V_m = Maximum value

ω= angular frequency

Example Problem:

If the peak value is 10V, find the instantaneous value at a point $\pi/4$ radians.

$V(t) = V_m \sin\omega t$

$= 10 \sin(\pi/4)$

$= 10 \times 0.707$

$= 7.07V$

2.3 INSTANTANEOUS VALUE

Consider the sine wave shown in figure 2.11

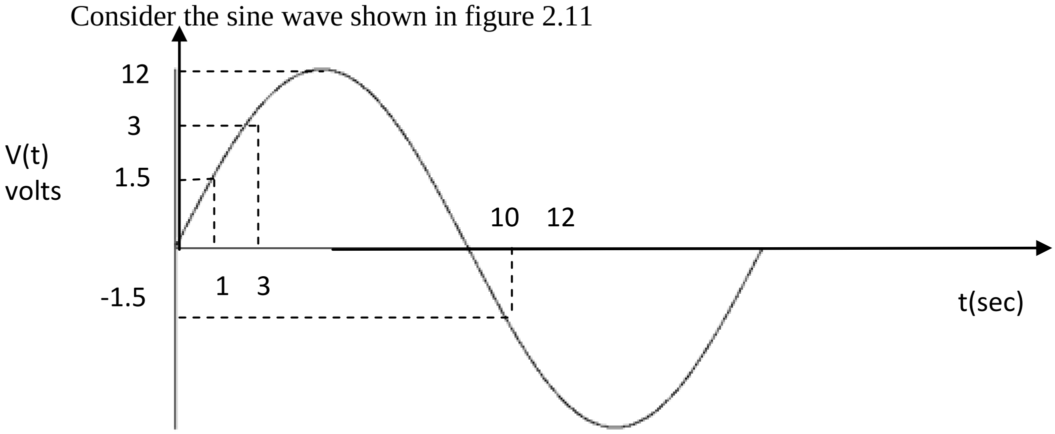


Fig.2.11

At any given time, it has some instantaneous value. The value varies continuously with respect to time. The value is different at different instant of time.

2.4 PEAK VALUE

The peak value of the sine wave is the maximum value of the sine wave during positive (or) negative half cycle. Since, the value of these two are equal in magnitude, a sine wave is characterized by a single peak value.

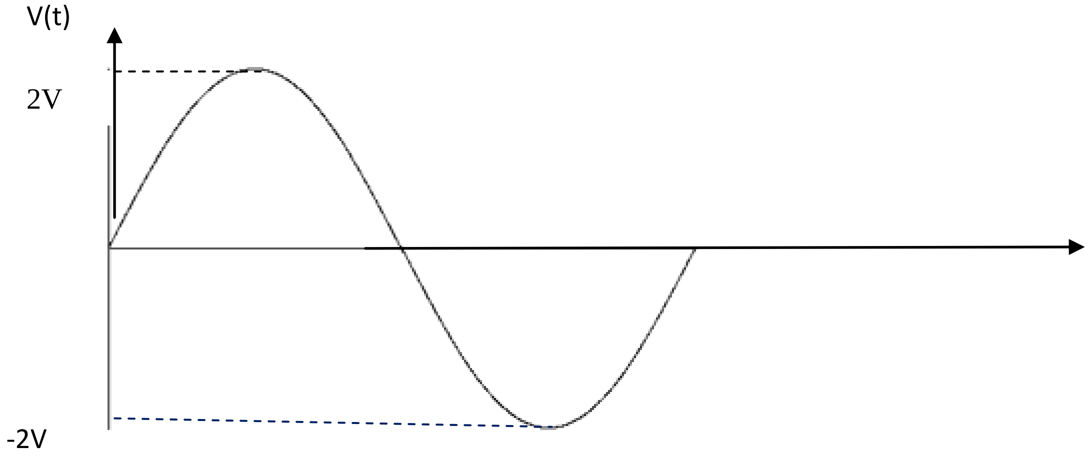


Fig.2.12

The peak value of the sine wave shown in figure is 2V.

2.5 PEAK TO PEAK VALUE

The peak to peak value of the sine wave is the value from the positive to the negative peak as shown in figure 2.13

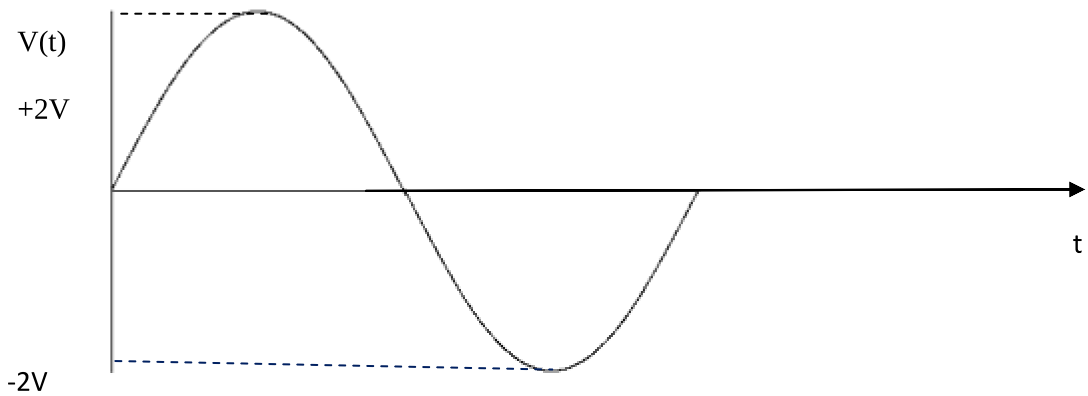


Fig.2.13

Here the peak to peak value is 4V

2.6 AVERAGE VALUE

The average value of a sine wave over one complete cycle is always zero. So, the average value of a sine wave is defined over a half cycle and not a full cycle period.

The average value of a sine wave is the total area under the half cycle curve divided by the distance of the curve. The average value of sine wave $V(t) = V_m \sin \omega t$ is given as follows:

$$V_{av} = \frac{1}{\pi} \int_0^{\pi} V_m \sin\omega t \, d(\omega t)$$

$$= \frac{1}{\pi} [-V_m \cos\omega t]_0^{\pi}$$

$$= \frac{2V_m}{\pi} = 0.637V_m$$

The average value of sine wave is shown by the dotted line in figure 2.14

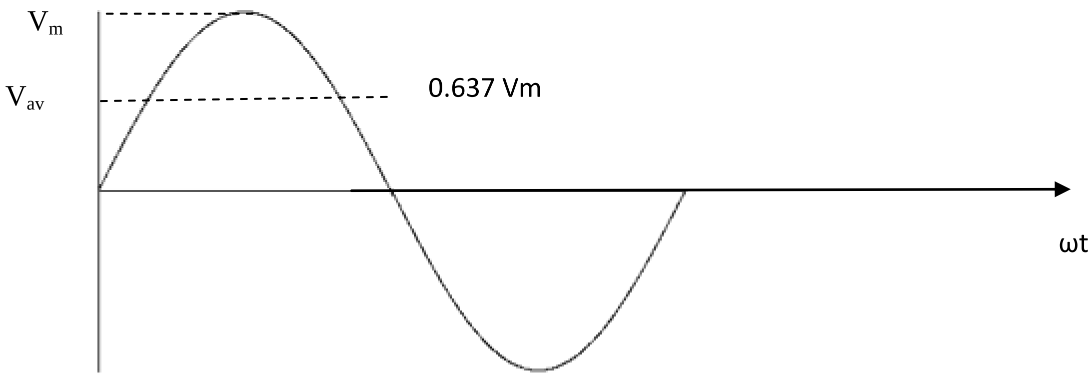


Fig.2.14

2.7 ROOT MEAN SQUARE (RMS) VALUE OF SINE WAVE(RMS

The root mean square value of a sine wave is a measure of the heating effect of the wave. When a resistor is connected across DC voltage source as shown in figure 2.15, a certain amount of heat is produced in the resistor in a given time.

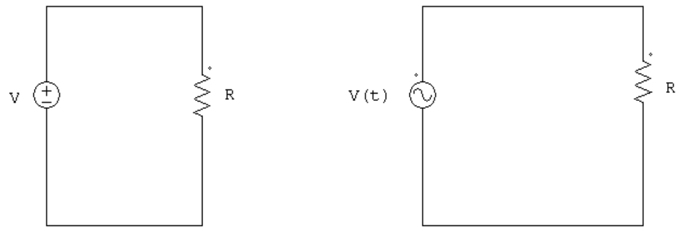


Fig.2.15

A similar resistor is connected across a AC voltage source for the same time. The value of the AC voltage is adjusted such that the same amount of heat is produced in the resistor in the case of DC source. This value is called RMS value.

The RMS value is given by

$$V_{rms} = \sqrt{\frac{1}{T} \int_o^T sqr\, V(t) 2dt}$$

$$= \sqrt{\frac{1}{T} \int_{o}^{T} (Vm \sin\omega t)2 \; d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi} \int_{o}^{2\pi} (Vm2\,[1 - \cos 2\omega t/2] \; d(\omega t)}$$

$$= \frac{Vm}{\sqrt{2}} = 0.707V_m$$

2.8 FORM FACTOR

Form factor of a waveform is defined as the ratio of RMS value to the average value of the wave.

Form factor = $\frac{\text{RMS value}}{\text{Averge value}}$

For the sinusoidal wave

$$\boxed{\text{Form factor} = \frac{0.707\ Vm}{0.637\ Vm} = 1.11}$$

2.9 PEAK FACTOR (CREST FACTOR)

The peak factor of a wave form is defined as the ratio of the peak value to the RMS value of the wave.

Peak factor = $\frac{\text{Vm}}{\text{V rms}}$

$= \frac{\text{Vm}}{\text{V rms}/\sqrt{2}}$

$$\boxed{\text{Peak factor} = 1.414}$$

It is also called as crest factor.

2.10 RECTANGULAR AND POLAR FORMS OF COMPLEX NUMBER

There are two basic forms of complex numbers notation. They are polar and rectangular. Rectangular form is where a complex number is denoted by its horizontal and vertical components. The horizontal component is referred to as the real component and the vertical component is referred to as the imaginary component.

Example Problem:

Let us represent the complex number p = x + iy in rectangular form as shown in figure 2.16.

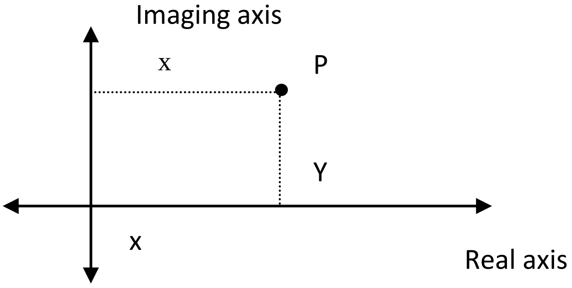


Fig.2.16

Polar form is where a complex number is denoted by the length and the angle of its vector. The length of the vector is absolute value (or) modules of the complex number. The angle with x axis is called the direction angle (or) argument of x + iy

Example Problem:

The complex number P is represented in polar form as shown in figure 2.17

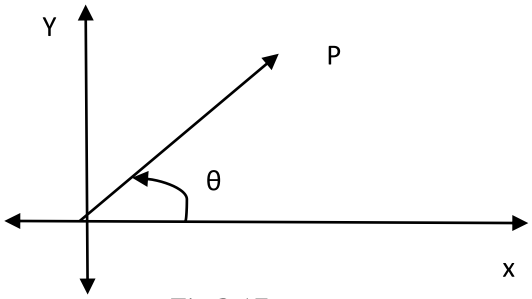


Fig.2.17

Complex number P is referred in polar form as $r \angle \theta$

Where r = distance between origin and point P. It is magnitude.

θ = angle between x axis and phasor.

Example Problem:

Convert 2+j8 into polar form.

Solution:

$R = \sqrt{x^2 + y^2}$

$= \sqrt{2^2 + 8^2}$

$= \sqrt{4 + 64}$

$= \sqrt{68}$

$= 8.2$

$\theta = \tan^{-1}(8/2)$

$= 76$

Example Problem :

Convert given polar to Rectangular form: $5\angle 30^0$

Solution:

$r = 5; \theta = 30^0$

$= 5 \text{ Cos } 30^0$

$= 4.3301$

$Y = r \sin \theta$

$= 5 \text{ Sin } 30^0 \quad = 2.5$

Answer = 4.33 + j 2.5

2.11 AC THROUGH PURE RESISTOR

Consider the circuit shown in figure 2.18

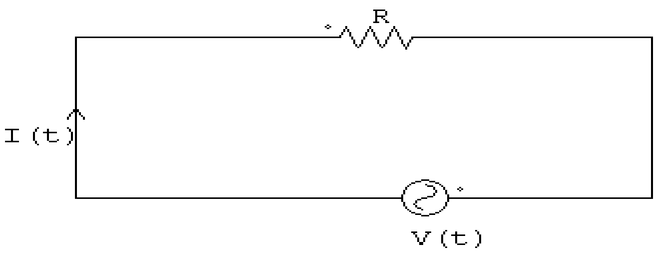


Fig.2.18

When a sinusoidal voltage V(t) is applied to a resistor, sinusoidal current I(t) passes through it. The voltage current relation in the case of resistor is linear.

As per ohm's law,

V(t) =i(t).R

we know, $V(t) = V_m \sin\omega t$

$I(t) = I_m \sin\omega t$

If, we substitute these in the above equation, we get

$V_m \sin\omega t = I_m R. \sin\omega t$

$V_m = I_m . R$

If we draw the wave form for both voltage and current as shown in figure. These is no phase difference between these two wave forms. Voltage and current is said to be in phase. Here the term impedance is defined as the ratio of voltage to current function. Impedance consists of magnitude and phase angle. Since, the phase difference is zero in the case of resistor, the phase angle is zero. The impedance consists only of magnitude.

Z = R

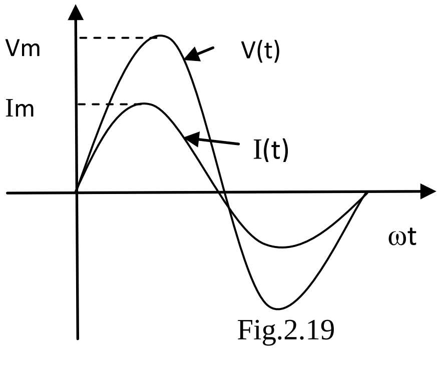


Fig.2.19

Wave form diagram

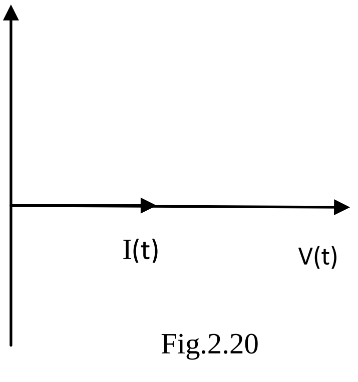


Fig.2.20

Phasor diagram

2.12 AC THROUGH PURE INDUCTOR

Consider the circuit shown in figure 2.21

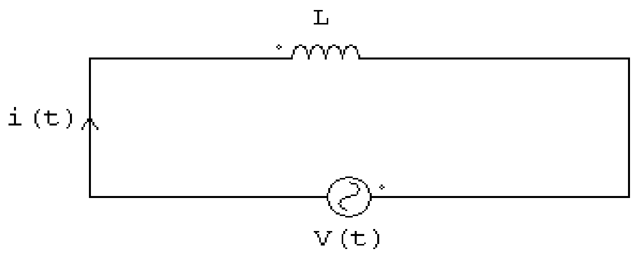


Fig.2.21

The voltage – current relation in the case of inductor is given by

$V(t) = L\frac{d(i)}{dt}$

Let $i(t) = I_m \sin\omega t$

substituting this in the above equation we get,

$V(t) = L\frac{d(I_m \sin\omega t)}{dt}$

$= L\omega I_m \cos\omega t$

$= \omega L I_m \cos\omega t$

$V_m \sin\omega t = \omega L I_m \cos\omega t$

$V_m \sin\omega t = \omega L I_m \sin(\omega t + 90^0)$

$\boxed{V_m = \omega L I_m = X_L I_M}$

$\sin\omega t \Longrightarrow \sin(\omega t + 90^0)$

$\boxed{\text{Inductive Reactance } X_L = \omega L}$

If we draw the both voltage and current as shown in figure 2.22. We can observe the phase difference 90^o between them.

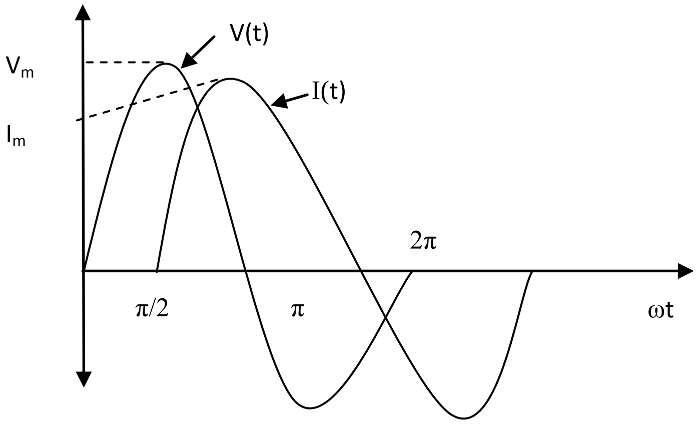


Fig.2.22

When sinusoidal current passes through the pure inductor, the current lags behind the voltage by 90^o as shown in figure 2.22. Impedance is given by

$$Z = \frac{V(t)}{I(t)} = \frac{Vm\sin(\omega t+90\theta)}{Im\sin\omega t}$$

Where $V_m = \omega L I_m$

$$= \frac{\omega L Im\sin(\omega t+90\theta)}{Im\sin\omega t}$$

$$\boxed{Z = X_L = \omega L}$$

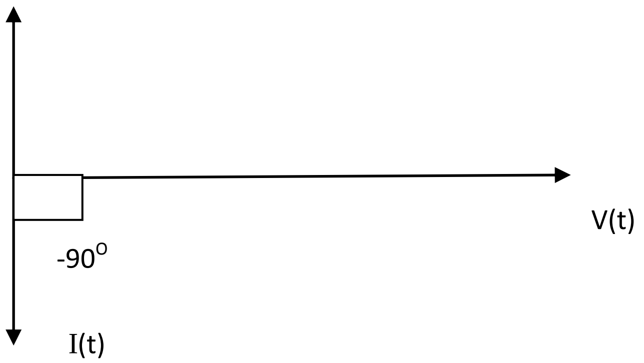


Fig.2.23

2.13 AC THROUGH PURE CAPACITOR

Consider the circuit shown in figure 2.24

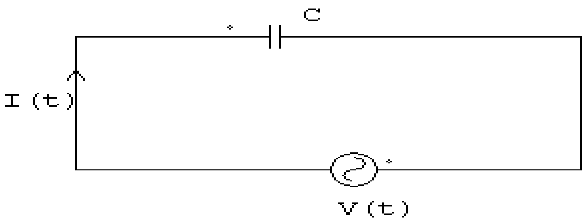


Fig.2.24

The voltage – current relationship in the case of pure capacitor is given by

$$V(t) = \frac{1}{c}\int i(t)dt$$

Let $i(t) = I_m \sin\omega t$

substituting this in the above equation, we get

$$V(t) = \frac{1}{c}\int Imsin\omega t\, dt$$

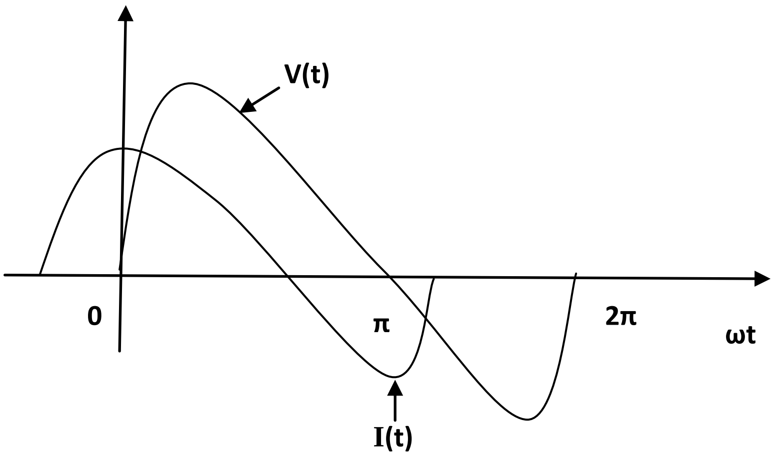
$$= \frac{1}{\omega c} I_m [-\cos\omega t]$$

$V(t) = \frac{Im}{\omega c} \sin(\omega t - 90^{\theta})$

$Vm = \frac{Im}{\omega c}$

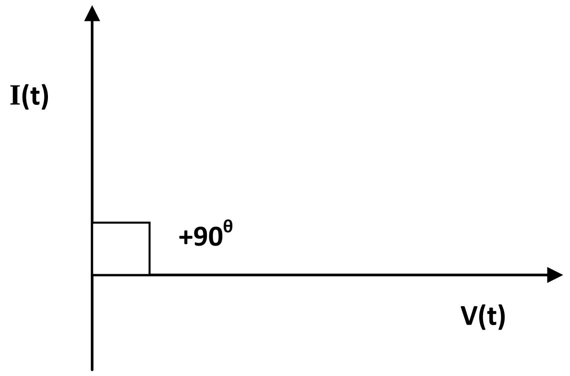
$$\boxed{Z = \frac{Vm}{Im} = \frac{1}{\omega c}}$$

If we draw the waveform for voltage and current as shown in figure. There is a phase difference between these two waveforms.



(i) wave form diagram

Fig.2.25



(ii) vector diagram

Fig.2.26

when AC passes through a pure capacitor, the current leads the voltage by 90^{θ}.
The impedance $Z = Xc = \frac{1}{\omega c}$

X_c is also called as capacitive reactance.

Example Problem:

A sinusoidal voltage is applied to a capacitor as shown in figure 2.24. The frequency of sine wave is 1KHZ. Determine the capacitive reactance.

$$X_c = \frac{1}{2\pi f c} = \frac{1}{2\pi \; X \; 1 X 1000 X 0.1 X 106}$$

$$= 1.59 \; K\Omega$$

2.14 CONCEPT OF IMPEDANCE

AC THROUGH RL CIRCUIT

Consider the RL circuit shown in figure 2.28

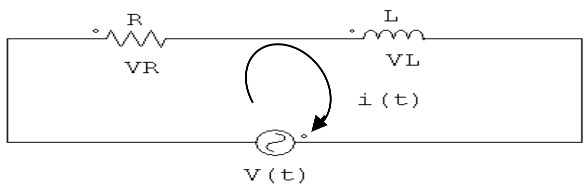


Fig.2.28

Applying Kirchoff's voltage law to the circuit shown in figure 2.28

We get

$V(t) = Ri(t) + L\frac{di(t)}{dt}$

If we apply complex function V(t)= Vm (cosω t + j sinωt) = $V_m e^{j\omega t}$ this circuit, the response may be

$i(t) = i_m e^{j\omega t}$

$V_m e^{j\omega t} = R\, i_m e^{jwt} + \frac{Ld(imej\omega t)}{dt}$

$V_m e^{j\omega t} = R\, i_m e^{jwt} + LI_m j\omega e^{j\omega t}$

$V_m = (R+j\omega L)I_m$

$V_m = \frac{VM}{(R+j\omega L)}$

We know

$I(t) = i_m e^{j\omega t}$

$= \frac{Vm}{(R+j\omega L)} \cdot e^{j\omega t}$

Impedance is defined as the ratio of voltage to current function,

$Z = \frac{V(t)}{I(t)} = \frac{Vm\ ej\omega t}{vm/R+j\omega Lej\omega t}$

$$\boxed{Z = R + j\omega L}$$

Complex impedance is the total opposition offered by the circuit elements to ac current and can be displayed on the complex plane. The impedance is denoted by Z. Here resistance R is the real part of the impedance and the reactance X_L is the imaginary part of impedance. The resistance R is located on the real axis. The inductive reactance X_L is located on the positive imaginary axis. The resultant of R and X_L is called complex impedance.

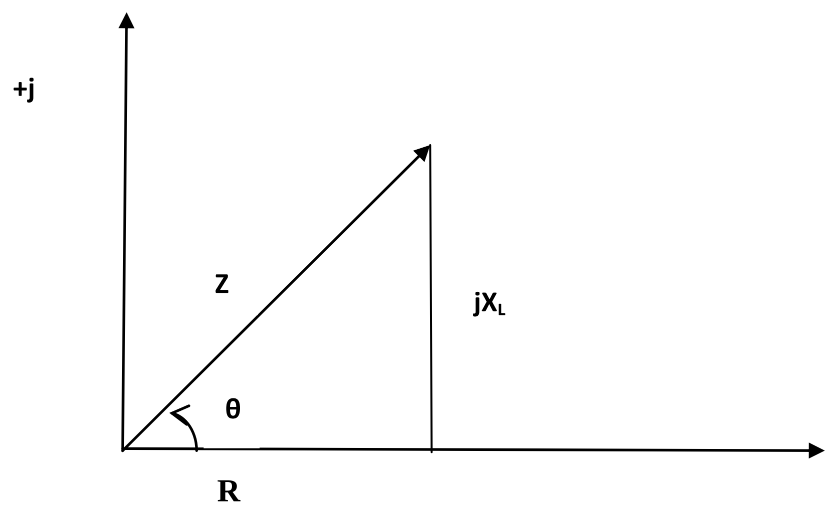


Fig.2.29

The impedance diagram for RL circuit is shown in figure. The impedance

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$Q = \tan^{-1}(\frac{\omega l}{R})$$

Impedance is the vector sum of the resistance and inductive reactance. The phasor diagram for series RL circuit is shown in figure 2.30

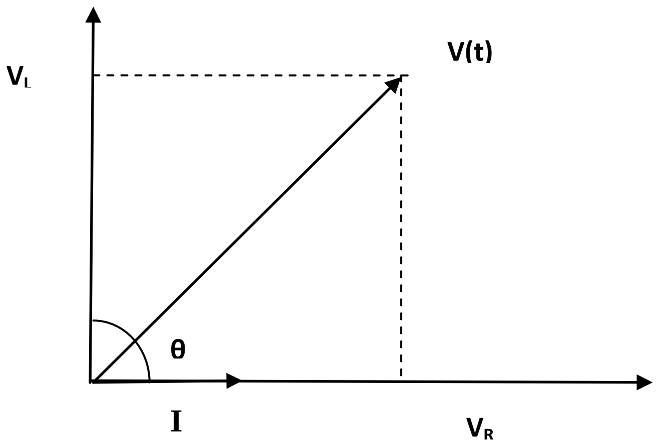


Fig.2.30

$$V = \sqrt{VR^2 + VL^2}$$
$$\theta = \tan^{-1}(\frac{Vl}{VR})$$

θ in the angle between Voltage and current.

2.15 AC THROUGH RC CIRCUIT

Consider the RC circuit shown in figure 2.31

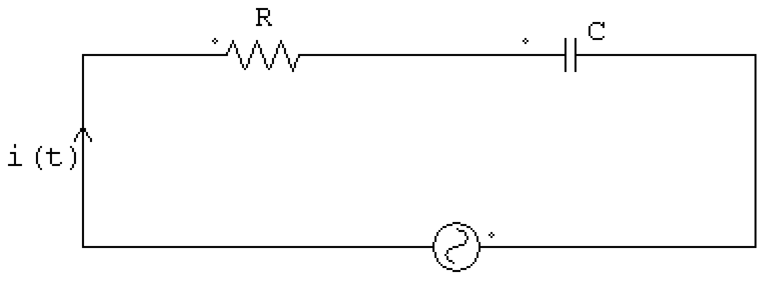


Fig.2.31

$$V(t)=V_m\, e^{jwt}$$

Applying Kirchoff's voltage law to the circuit is shown in figure 2.31

$$V(t) = R.i(t) + \frac{1}{C}\int i(t)\, dt$$

If we apply complex function $V(t) = V_m e^{j\omega t}$ to the circuit, the response may be $i(t) = I_m e^{j\omega t}$. Substituting this in the above equation,

$$V_m e^{j\omega t} = R.I_m e^{j\omega t} + \frac{1}{C}\int I_m e^{j\omega t}\, dt$$

$$= R.I_m e^{j\omega t} + \frac{1}{C}\frac{Im}{j\omega}(e^{j\omega t})$$

$$= R.I_m e^{j\omega t} - \frac{jIm}{\omega c}(e^{j\omega t})$$

$$= [R.I_m - \frac{jIm}{\omega c}](e^{j\omega t})$$

$$V_m = (R - \frac{j}{\omega c})\, I_m$$

$$Z = \frac{Vm}{Im} = R = j/\omega C$$

$Z = R - j/\omega C$ or $Z = R + 1/j\omega C$

The vector diagram is shown in figure 2.32.

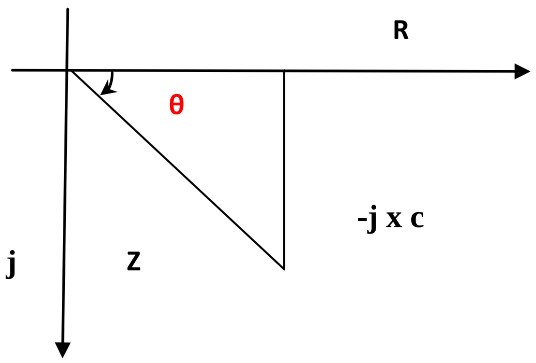


Fig.2.32

Here impedance Z consists of resistance(R), which is real part and capacitive reactance $X_c = 1/j\omega c$, which is imaginary part of the impedance. The resistance R is located on the real axis, and the capacitive reactance X_c is located on the J axis in impedance diagram.

$$Z = \sqrt{R^2 + Xc^2} \quad \text{(or)} \quad Z = \sqrt{R^2 + (\tfrac{1}{j\omega c})^2}$$

$$Q = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

Z is the sum of resistance and capacitive reactance. The angle between resistance and impedance is the phase angle between applied voltage and current in the circuit.

The phasor diagram for RC series circuit is shown in figure 2.33 and 2.34.

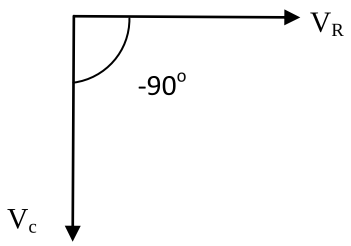


Fig.2.33

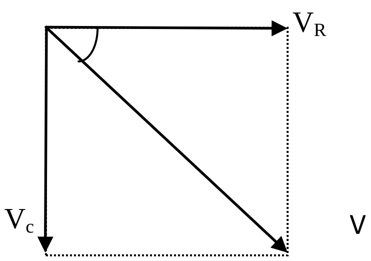


Fig.2.34

The resistor voltage is in phase with the current and the capacitor voltage lags behind the current by 90^o. From Kirchoff's voltage law, it can be written as

$$V = \sqrt{Vr^2 + Vc^2}$$

The phase angle between the resistor voltage source voltage is

$$\boldsymbol{\theta} = \tan^{-1}(V_c/V_R)$$

2.16 AC THROUGH R-L-C SERIES CIRCUIT

Consider the R-L-C circuit shown in figure 2.35

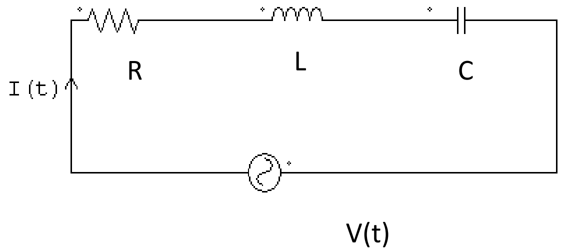


Fig.2.35

A series RLC Circuit is the series combination of Resistance, Inductance and Capacitance. If we observe the Impedance diagrams of series RL and series RC Circuits as shown in Figure 2.36 and 2.37 the Inductive reactance X_L is displayed on the +j axis and the Capacitive reactance X_C is displayed on the –j axis. These reactances are 180° apart and tend to cancel each other.

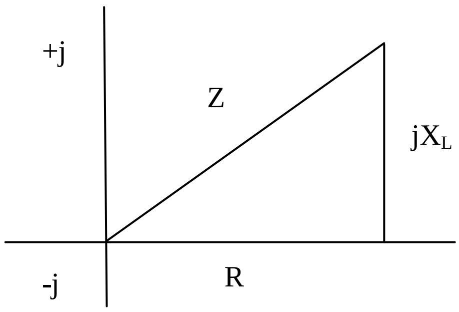


Fig.2.36

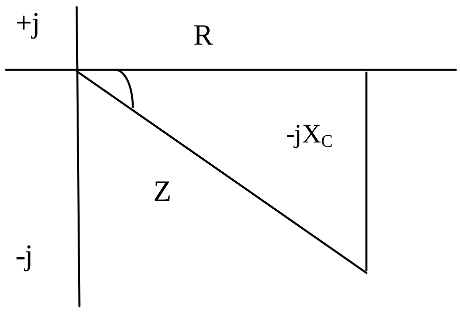


Fig.2.37

The magnitude and type of reactance in a series RLC circuit is the difference between the two reactances. The Impedance for an RLC Series Circuit is given by

$$Z = \sqrt{R^2 + (XL - XC)^2}$$

Phase angle of RLC Circuit $\quad \theta = \tan^{-1}\dfrac{XL - XC}{R}$

Phasor diagram:

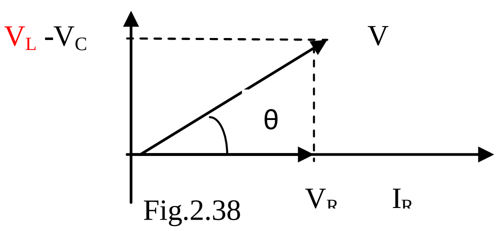


Fig.2.38

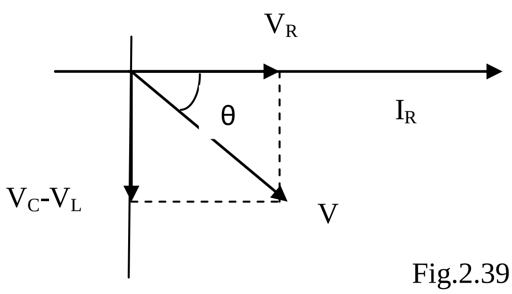


Fig.2.39

The equations and phasor diagram are shown :

$V_R = IR$

$V_L = IX_L$
$V_C = IX_C$

By applying Kirchoff's law

$V = V_R+jV_L-jV_c$

$= IR+jIX_L-jIX_C$

$= I[R+j(X_L-X_C]$

$=IZ$

Where $Z = \sqrt{R^2 + (XL - XC)^2}$

$\theta = \tan^{-1}\dfrac{(XL - XC)}{R}$

2.17 POWER IN AC CIRCUIT

In a pure Resistive Circuit, all the power delivered by the source is dissipated in the form of heat by the Resistance. In a pure Reactive Circuits energy delivered by the source is stored by the Inductor (or) Capacitor in magnetic (or) electric field and returned to the source. So that no net energy is transferred. When there is a Complex Impedance in a circuit, part of energy is alternately stored and returned by the reactive part, and part of the power is dissipated by resistive part.

Instantaneous power is the power at any instant of time. Let us consider the voltage, current and power wave forms shown in figure 2.40 and 2.41.

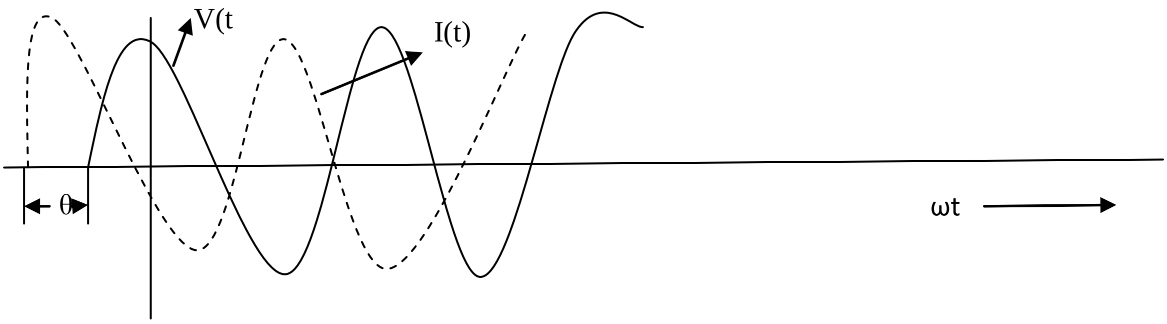


Fig.2.40

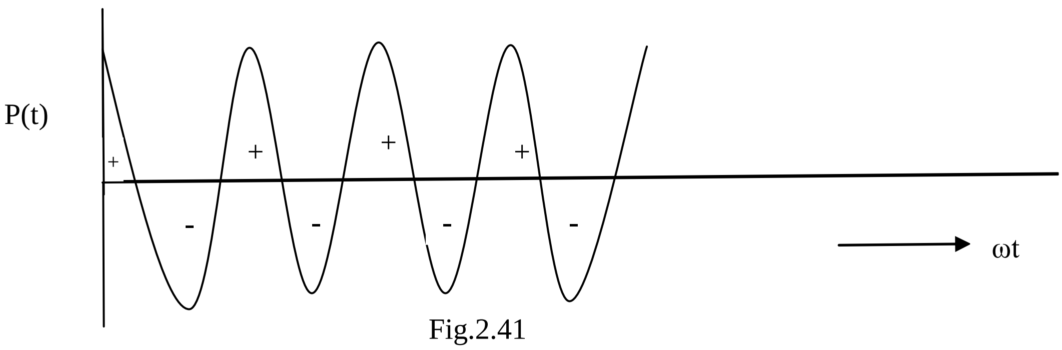


Fig.2.41

The positive portion of the power cycle varies with the phase angle between the voltage and current waveforms. If the circuit is pure resistive, the phase angle between voltage and current is zero then there is no negative cycle in the power curve. Hence all the power delivered by source is dissipated by Resistance.

When the phase angle between voltage and current increases the negative portion of power cycles increase and the lesser power is dissipated. When phase angle becomes $\pi/2$, the positive and negative portions of the power cycle are equal. At this instant power dissipation is zero. Then the power delivered to the load is returned to the source.

2.17.1 Average power:

To get average power, called as true or real power, we have to take the product of the effective values of voltage and current multiplied by the cosine of the phase angle between the voltage and current.

Average power = $V_{eff} I_{eff} \cos\theta$ The Unit is Watts, W.

If we consider a pure resistor circuit, the phase angle between voltage and current is zero. Hence the Average Power is $P = I_M^2 R / 2$

If we consider a pure reactive circuit, the phase angle between voltage and current is 90^0.Hence the Average power is zero. $P_{av} = 0$

If the circuit contains complex Impedance, the average power is the power dissipated in the resistive part only.

2.17.2 Apparent Power:

If we consider a sinusoidal voltage applied to the circuit, the product of voltage and current is called Apparent Power. The apparent power is expressed in volt ampere. (VA)

Apparent Power = $V_{eff} I_{eff}$

2.17.3 Reactive Power:

The power stored by reactive component is called Reactive Power, in VAR.

Power Triangle:

The Power Triangle is Shown in Figure 2.42

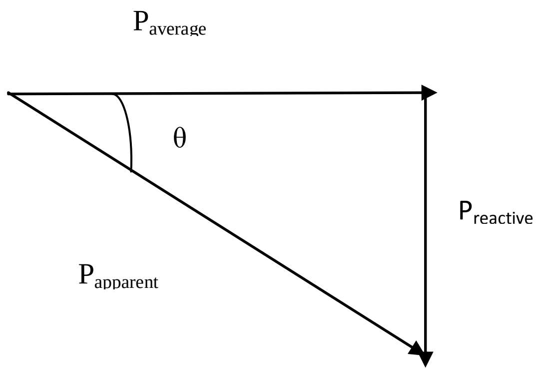


Fig.2.42

2.18 Power factor:

The power factor is useful in determining true power, transferred to the Load. It varies from 0 to 1. The highest value of power factor is 1. This indicates that the current to a load is in phase with voltage across it. When the power factor is zero, the current to a load is out of phase with voltage across it.

Average Power = Apparent Power X Power Factor

Power Factor = $\frac{\text{Average Power}}{\text{Apparent Power}}$

Power factor is defined as the cosine of the phase angle between voltage and current.

Power factor = Cos θ = $\frac{R}{Z}$

Example Problem:

A 10Ω Resistor is connected in series with an Inductor having X_L = 5 Ω. Find the power factor given data R = 10 Ω, X_L = 5 Ω

Impedance = Z = $\sqrt{R^2 + XL^2}$

$= \sqrt{(10)^2 + (5)^2}$

$= \sqrt{100 + 25} = \sqrt{125} = 11.18\ \Omega$

Power Factor $= \frac{R}{Z}$

$= \frac{10}{11.18} = 0.89$

Example Problem:

A series circuit has R = 10 Ω, L = 50mH and C = 100μF and is supplied with 200V, 50Hz. Find (a) the impedance (b) the current, (c) the power (d) the power factor.

$X_L = 2\pi fL = 15.7$ ohms

$X_c = \frac{1}{2\pi fC} = 31.8\ \Omega$

$Z = [R+j(X_L-X_C)]$

$= 10 - j16.1 = 18.95\ \angle{-58^0}$

(a) Impedance = 18.95ohms

(b) Current = $I = \frac{200}{18.95} = 10.55$ Amp

(c) Power $= I^2R = 1113$ watts

(d) Power Factor = $\cos 58^0 = 0.53$

Power factor $= \frac{R}{Z} = \frac{10}{18.95} = 0.53$

Power = $V.I \cos\theta$

$= 200 \times 10.55 \times 0.53$

2.2 INTRODUCTION TO HARMONICS

In an AC circuit, a resistance behaves in exactly the same way as it does in a DC circuit. That is, the current flowing through the resistance is proportional to the voltage across it.

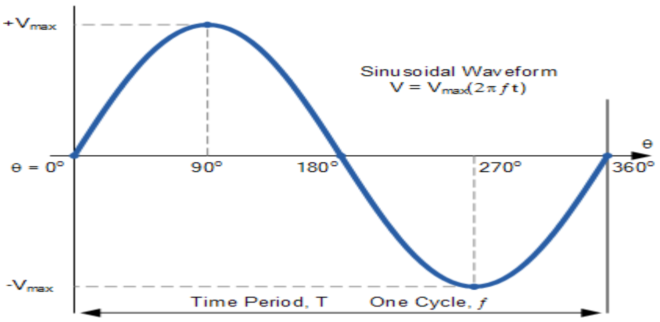


Fig.2.43

This is because a resistor is a linear device and if the voltage applied to it is a sine wave, the current flowing through it is also a sine wave so the phase difference between the two sinusoids is zero.

Generally when dealing with alternating voltages and currents in electrical circuits it is assumed that they are pure and sinusoidal in shape with only one frequency value, called the “fundamental frequency” being present, but this is not always the case.

In an electrical or electronic device or circuit that has a voltage-current characteristic which is not linear, that is, the current flowing through it is not proportional to the applied voltage. The alternating waveforms associated with the device will be different to a greater or lesser extent to those of an ideal sinusoidal waveform. These types of waveforms are commonly referred to as non-sinusoidal or complex waveforms.

Complex waveforms are generated by common electrical devices such as iron-cored inductors, switching transformers, electronic ballasts in fluorescent lights and other such heavily inductive loads as well as the output voltage and current waveforms of AC alternators, generators and other such electrical machines. The result is that the current waveform may not be sinusoidal even though the voltage waveform is sinusoidal.

Also most electronic power supply switching circuits such as rectifiers, silicon controlled rectifier (SCRs), power transistors, power converters and other such solid state switches which cut and chop the sinusoidal waveform of power supplies to control motor power, or to convert the sinusoidal AC supply to DC. Theses switching circuits tend to draw current only at the peak values of the AC supply and since the switching current waveform is non-sinusoidal the resulting load current is said to contain **Harmonics**.

Non-sinusoidal complex waveforms are constructed by "adding" together a series of sine wave frequencies known as "Harmonics". Harmonics is the generalised term used to describe the distortion of a sinusoidal waveform by waveforms of different frequencies.

Then whatever its shape, a complex waveform can be split up mathematically into its individual components called the fundamental frequency and a number of "harmonic frequencies".

Fundamental Frequency

A **Fundamental Waveform** (or first harmonic) is the sinusoidal waveform that has the supply frequency. The fundamental is the lowest or base frequency, f, on which the complex waveform is built and as such the periodic time, T of the resulting complex waveform will be equal to the periodic time of the fundamental frequency.

Let's consider the basic fundamental or 1st harmonic AC waveform as shown.

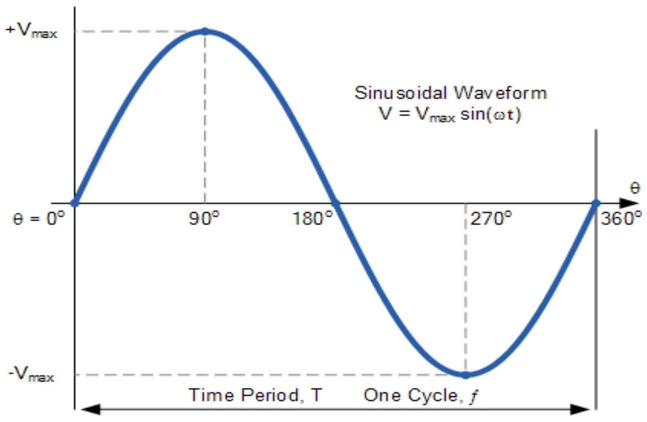


Fig.2.44

Where, Vmax is the peak value in volts and f is the waveform's frequency in Hertz (Hz).

We can see that a sinusoidal waveform is an alternating voltage (or current), which varies as a sine function of angle, $2\pi f$. The waveform's frequency, f, is determined by the number of cycles per second. In United Kingdom this fundamental frequency is set at 50Hz while in the United States it is 60Hz.

Harmonics are voltages or currents that operate at a frequency that is an integer (whole-number) multiple of the fundamental frequency. So given a 50Hz fundamental waveform, this means a 2nd harmonic frequency would be 100Hz (2 x 50Hz), a 3rd harmonic would be 150Hz (3 x 50Hz), a 5th at 250Hz, a 7th at 350Hz and so on. Likewise, given a 60Hz fundamental waveform, the 2nd, 3rd, 4th and 5th harmonic frequencies would be at 120Hz, 180Hz, 240Hz and 300Hz respectively.

So in other words, we can say that "harmonics" are multiples of the fundamental frequency and can therefore be expressed as: $2f$, $3f$, $4f$, etc. as shown.

Complex Waveforms Due To Harmonics

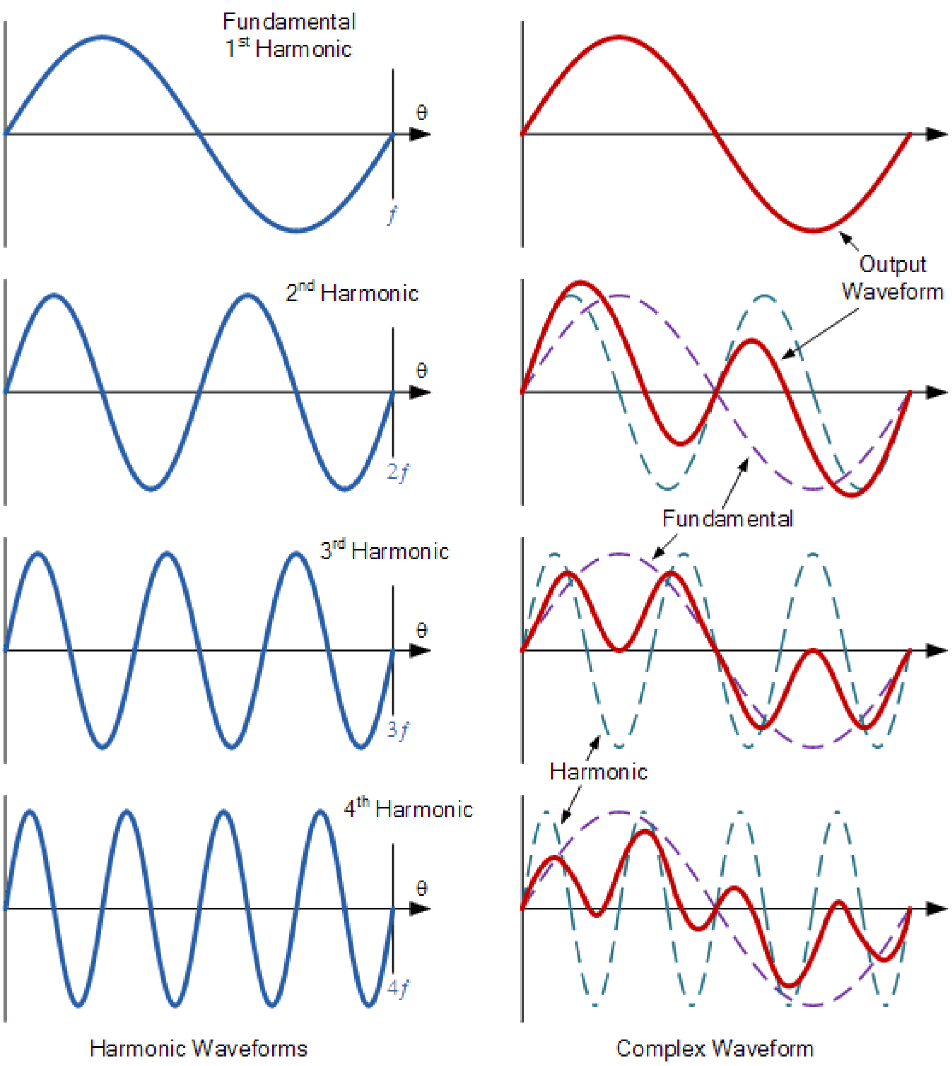


Fig.2.45

Note that the red waveforms, shown above, are the actual shapes of the waveforms as seen by a load due to the harmonic content being added to the fundamental frequency.

The fundamental waveform can also be called a 1^{st} harmonics waveform. Therefore, a second harmonic has a frequency twice that of the fundamental, the third harmonic has a frequency three times the fundamental and a fourth harmonic has one four times the fundamental as shown in the left hand side column.

The right hand side column shows the complex wave shape generated as a result of the effect between the addition of the fundamental waveform and the harmonic waveforms at different harmonic frequencies. Note that the shape of the resulting complex waveform will depend not only on the number and amplitude of the harmonic frequencies present, but also on the phase relationship between the fundamental or base frequency and the individual harmonic frequencies.

We can see that a complex wave is made up of a fundamental waveform plus harmonics, each with its own peak value and phase angle. For example, if the fundamental frequency is given as; $E = V_{max}(2\pi f t)$, the values of the harmonics will be given as:

For a second harmonic:

$E_2 = V_{2max}(2\times2\pi f t) = V_{2max}(4\pi f t), = V_{2max}(2\omega t)$

For a third harmonic:

$E_3 = V_{3max}(3\times2\pi f t) = V_{3max}(6\pi f t), = V_{3max}(3\omega t)$

For a fourth harmonic:

$E_4 = V_{4max}(4\times2\pi f t) = V_{4max}(8\pi f t), = V_{4max}(4\omega t)$

and so on.

Then the equation given for the value of a complex waveform will be:

$$E_T = E_1 + E_2 + E_3 + \ldots\ldots E_{(n)} \quad \text{etc.}$$

$$E_T = V_{1max}\sin(2\pi f \tau) + V_{2max}\sin(4\pi f \tau) + V_{3max}\sin(6\pi f \tau) \ldots \text{etc.}$$

Harmonics are generally classified by their name and frequency, for example, a 2^{nd}harmonic of the fundamental frequency at 100 Hz, and also by their sequence. Harmonic sequence refers to the phasor rotation of the harmonic voltages and currents with respect to the fundamental waveform in a balanced, 3-phase 4-wire system.

A positive sequence harmonic (4th, 7th, 10th, …) would rotate in the same direction (forward) as the fundamental frequency. Where as a negative sequence harmonic (2nd, 5th, 8th, …) rotates in the opposite direction (reverse) of the fundamental frequency.

Generally, positive sequence harmonics are undesirable because they are responsible for overheating of conductors, power lines and transformers due to the addition of the waveforms.

Negative sequence harmonics on the other hand circulate between the phases creating additional problems with motors as the opposite phasor rotation weakens the rotating magnetic field require by motors, and especially induction motors, causing them to produce less mechanical torque.

Another set of special harmonics called "triplens" (multiple of three) have a zero rotational sequence. *Triplens* are multiples of the third harmonic (3rd, 6th, 9th, …), etc, hence their name, and are therefore displaced by zero degrees. Zero sequence harmonics circulate between the phase and neutral or ground.

Unlike the positive and negative sequence harmonic currents that cancel each other out, third order or triplen harmonics do not cancel out. Instead add up arithmetically in the common neutral wire which is subjected to currents from all three phases.

The result is that current amplitude in the neutral wire due to these triplen harmonics could be upto 3 times the amplitude of the phase current at the fundamental frequency causing it to become less efficient and overheat.

Then we can summarise the sequence effects as multiples of the fundamental frequency of 50Hz as:

Harmonic currents can produce a number of problems:

1. Equipment heating
2. Equipment malfunction
3. Equipment failure
4. Communications interference
5. Fuse and breaker mis-operation
6. Process problems
7. Conductor heating.

Types of equipment that generate harmonics:

Harmonic load currents are generated by all non-linear loads. These include:

For Single phase loads, e.g.

Switched mode power supplies (SMPS)

Electronic fluorescent lighting ballasts

Compact fluorescent lamps (CFL)

Small uninterruptible power supplies (UPS) units

For Three phase loads, e.g.

Variable speed drives

Large UPS units

REVIEW QUESTIONS – UNIT II

2 Mark Questions:

1. Define period of sine wave
2. Define frequency of sine wave
3. Find the frequency of sine wave When period is 1msec
4. What is phase of sine wave?
5. If the peak value is 20V, find the instantaneous value at a point $\pi/4$ radians.
6. Define form factor
7. Define peak factor
8. A sinusoidal voltage is applied to a capacitor. The frequency of sine wave is 2 KHZ. Determine the capacitive reactance
9. What is average power?
10. What is apparent power?

3 Mark Questions:

1. Explain average value
2. Explain RMS value
3. What is impedance?
4. Draw the vector diagram for impedance of RL series circuit
5. Draw the phasor diagram for impedance of RC series circuit

10 Mark questions:

1. Explain with phasor diagram the concept of AC through pure resistor , Inductor and capacitor
2. Derive the expression for the energy stored in capacitor
3. With phasor diagram Explain in detail RL series circuit
4. With phasor diagram Explain in detail RC series circuit
5. With phasor diagram explain in detail RLC series circuit

UNIT III
RESONANCE AND 3Φ AC CIRCUITS

3.1 RESONANCE

An a.c. circuit is said to be in resonance when the circuit power factor is unity. i.e., At $X_L = X_C$ resonance occurs. In a resonant circuit resistors, inductors and capacitors are connected either in series or parallel. This circuit acts as a pure resistive circuit.

The frequency at which resonance occurs is called the resonant frequency (f_r).

3.2 Series Resonance Circuit

In a series RLC circuit there becomes a frequency point were the inductive reactance of the inductor becomes equal in value to the capacitive reactance of the capacitor. In other words, $X_L = X_C$. The point at which this occurs is called the **Resonant Frequency** point, (f_r) of the circuit, and as we are analysing a series RLC circuit this resonance frequency produces a **Series Resonance**.

Series Resonance circuits are one of the most important circuits used electrical and electronic circuits. They can be found in various forms such as in AC mains filters, noise filters and also in radio and television tuning circuits producing a very selective tuning circuit for the receiving of the different frequency channels. Consider the simple series RLC circuit below.

Series RLC Circuit

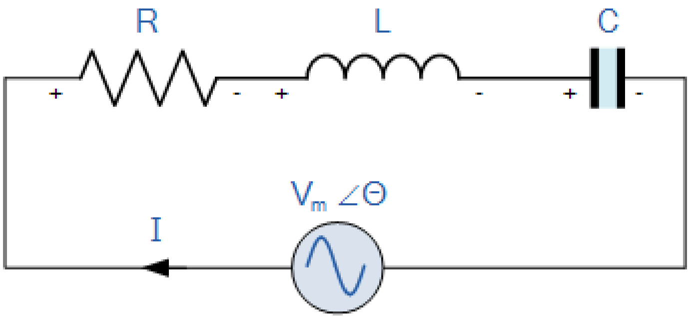


Fig 3.1

First, let us define what we already know about series RLC circuits.

- Inductive reactance : $X_L = 2\pi f L = \omega L$
- Capacitive reactance : $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{\omega C}$
- When $X_L > X_C$ the circuit is Inductive
- When $X_C > X_L$ the circuit is Capacitive
- Total circuit reactance = $X_T = X_L - X_C$ or $X_C - X_L$
- Total circuit impedance = $Z = \sqrt{R^2 + X_T^2} = R + jX$

From the above equation for inductive reactance, if either the **Frequency** or the **Inductance** is increased the overall inductive reactance value of the inductor would also increase. As the frequency approaches infinity the inductive reactance would also increase towards infinity with the circuit element acting like an open circuit.

However, as the frequency approaches zero or DC, the inductors reactance would decrease to zero, causing the opposite effect acting like a short circuit. This means then that inductive reactance is "**Proportional**" to frequency and is small at low frequencies and high at higher frequencies and this is demonstrated in the following curve:

3.2.1 Series Resonance Frequency

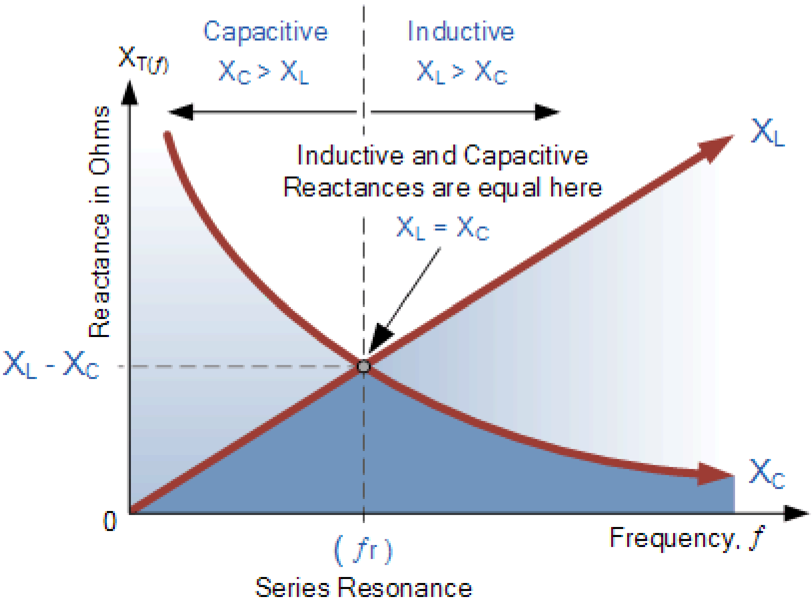


Fig 3.2

where: f_r is in Hertz, L is in Henries and C is in Farads.

Electrical resonance occurs in an AC circuit when the two reactances which are opposite and equal cancel each other out as $X_L = X_C$ and the point on the graph at which this happens were the two reactance curves cross each other. In a series resonant circuit, the resonant frequency, f_r point can be calculated as follows.

$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \ f_r = \frac{1}{2\pi\sqrt{LC}} \ (\mathrm{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}} \ (\mathrm{rads})$$

At resonance, the two reactances cancel each other out thereby making a series LC combination act as a short circuit with the only opposition to current flow in a series resonance circuit being the resistance, R. In complex form, the resonant frequency is the frequency at which the total impedance of a series RLC circuit becomes purely "real", that is no imaginary impedance's exist. This is because at resonance they are cancelled out. So the total impedance of the series circuit becomes just the value of the resistance and therefore: Z = R.

Then at resonance the impedance of the series circuit is at its minimum value and equal only to the resistance, R of the circuit. The circuit impedance at resonance is called the "dynamic impedance" of the circuit and depending upon the frequency, X_C (typically at high frequencies) or X_L (typically at low frequencies) will dominate either side of resonance as shown below.

3.2.2 Impedance in a Series Resonance Circuit

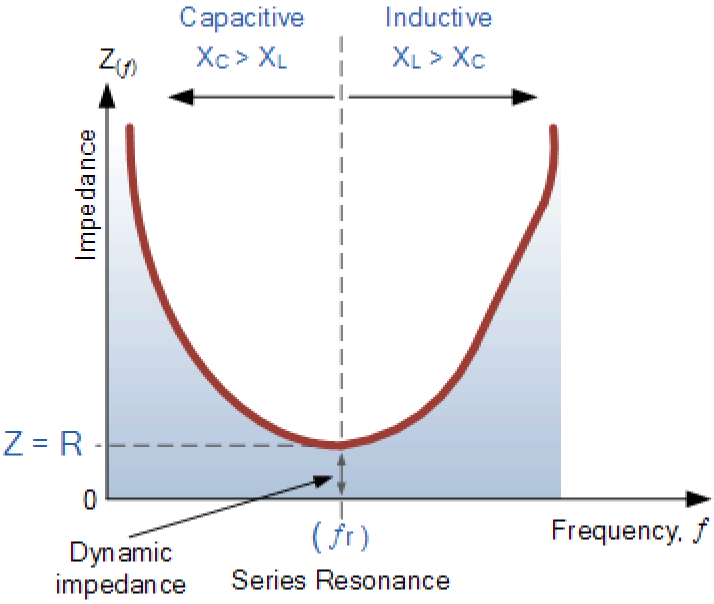


Fig 3.3

Note that when the capacitive reactance dominates the circuit the impedance curve has a hyperbolic shape to itself, but when the inductive reactance dominates the circuit the curve is non-symmetrical due to the linear response of X_L. You may also note that if the circuit impedance is at its minimum at resonance then consequently, the circuit **admittance** must be at its maximum and one of the characteristics of a series resonance circuit is that admittance is very high. But this can be a bad thing because a very low value of resistance at resonance means that the resulting current flowing through the circuit may be dangerously high.

We recall from the previous tutorial about series RLC circuits that the voltage across a series combination is the phasor sum of V_R, V_L and V_C. Then if at resonance the two reactances are equal and cancelling, the two voltages representing V_L and V_C must also be opposite and equal in value thereby cancelling each other out because with pure components the phasor voltages are drawn at $+90^o$ and -90^o respectively.

Then in a **series resonance** circuit as $V_L = -V_C$ the resulting reactive voltages are zero and all the supply voltage is dropped across the resistor. Therefore, $V_R = V_{supply}$ and it is for this reason that series resonance circuits are known as voltage resonance circuits, (as opposed to parallel resonance circuits which are current resonance circuits).

3.2.3 Series RLC Circuit at Resonance

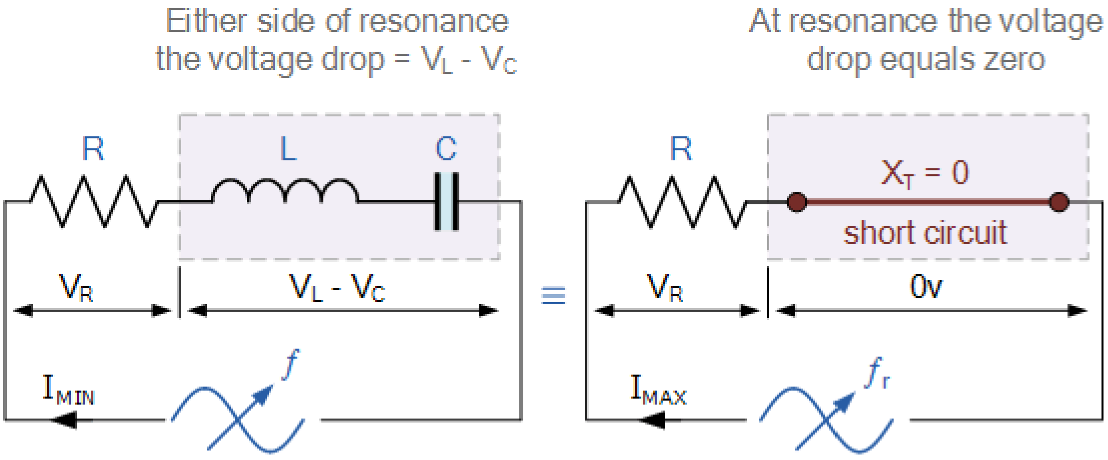


Fig 3.4

Since the current flowing through a series resonance circuit is the product of voltage divided by impedance, at resonance the impedance, Z is at its minimum value, (=R). Therefore, the circuit current at this frequency will be at its maximum value of V/R as shown below.

3.2.4 Series Circuit Current at Resonance

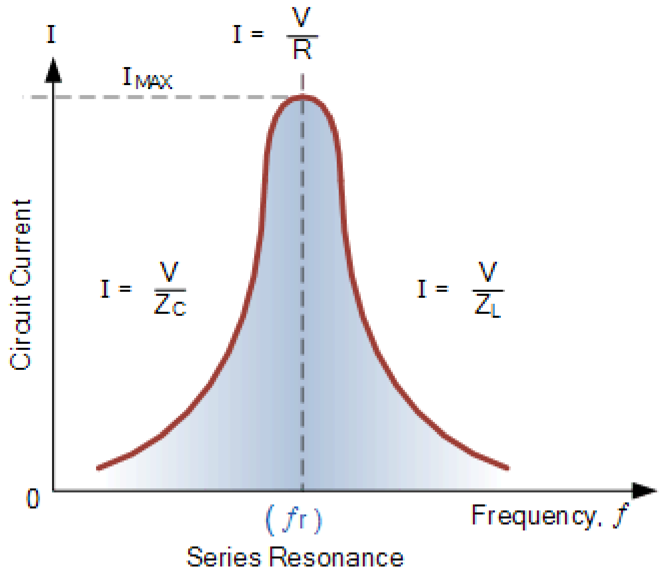


Fig 3.5

The frequency response curve of a series resonance circuit shows that the magnitude of the current is a function of frequency and plotting this onto a graph shows us that the response starts at near to zero, reaches maximum value at the resonance frequency when $I_{MAX} = I_R$ and then drops again to nearly zero as f becomes infinite. The result of this is that the magnitudes of the voltages across the inductor, L and the capacitor, C can become many times larger than the supply voltage, even at resonance but as they are equal and at opposition they cancel each other out.

The maximum current through the circuit at resonance is limited only by the value of the resistance (a pure and real value), the source voltage and circuit current must therefore be in phase with each other at this frequency. Then the phase angle between the voltage and current of a series resonance circuit is also a function of frequency for a fixed supply voltage and which is zero at the resonant frequency point when: V, I and V_R are all in phase with each other as shown below. Consequently, if the phase angle is zero then the power factor must therefore be unity.

3.2.5 Phase Angle of a Series Resonance Circuit

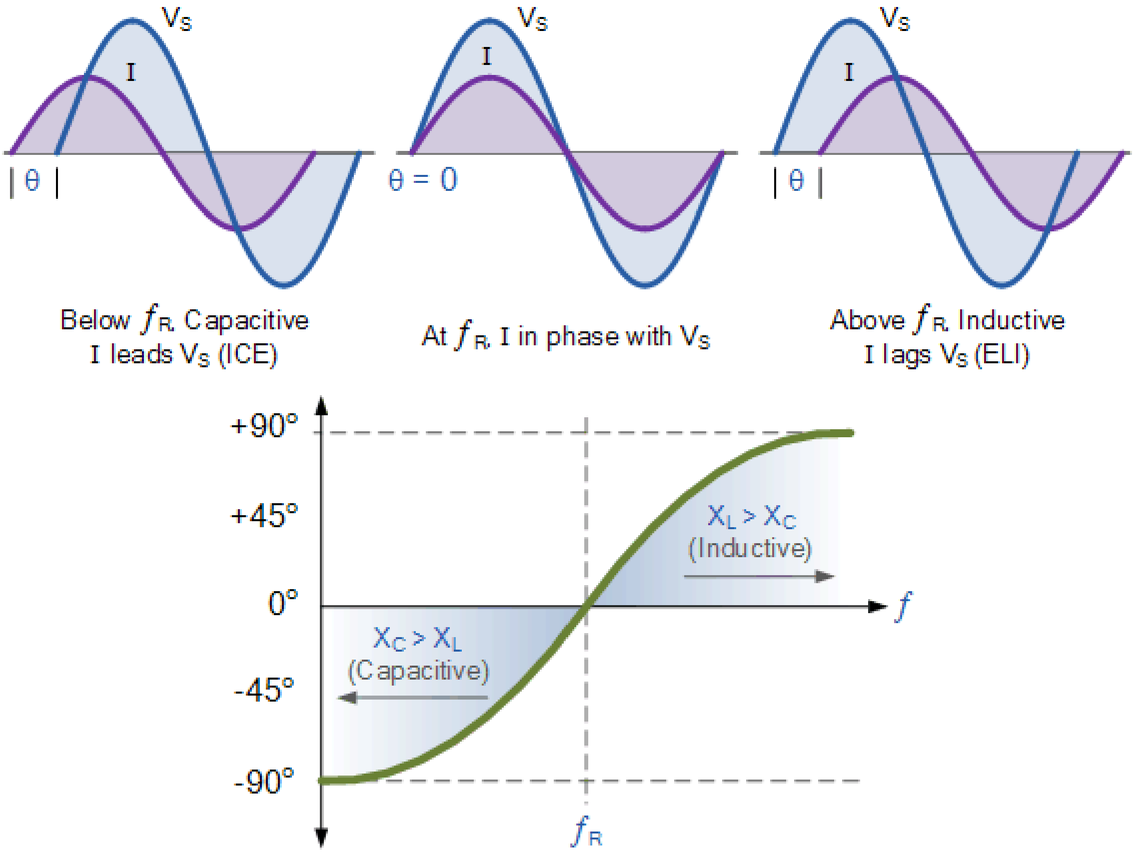


Fig 3.6

Notice also, that the phase angle is positive for frequencies above f_r and negative for frequencies below f_r and this can be proven by,

$$\arctan = \frac{X_L - X_C}{R} = 0^o \quad (\text{all real})$$

3.2.6 Bandwidth of a Series Resonance Circuit

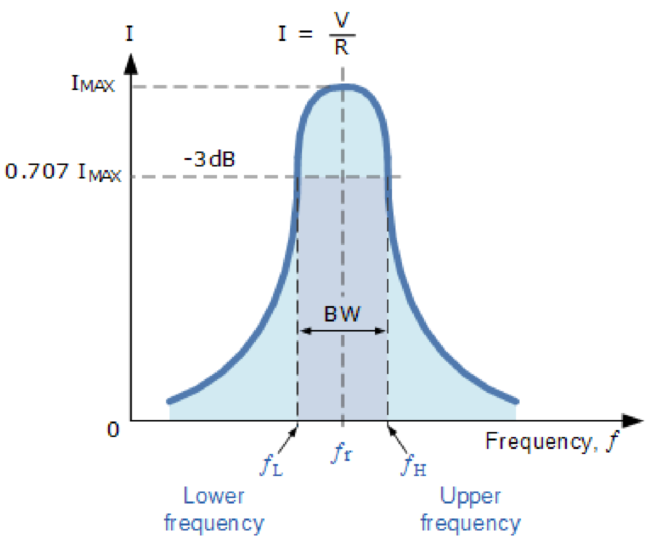


Fig 3.7

The frequency response of the circuits current magnitude above, relates to the "sharpness" of the resonance in a series resonance circuit. The sharpness of the peak is measured quantitatively and is called the **Quality factor, Q** of the circuit. The quality factor relates the maximum or peak energy stored in the circuit (the reactance) to the energy dissipated (the resistance) during each cycle of oscillation meaning that it is a ratio of resonant frequency to bandwidth and the higher the circuit Q, the smaller the bandwidth, $Q = f_r/BW$.

As the bandwidth is taken between the two -3dB points, the **selectivity** of the circuit is a measure of its ability to reject any frequencies either side of these points. A more selective circuit will have a narrower bandwidth whereas a less selective circuit will have a wider bandwidth. The selectivity of a series resonance circuit can be controlled by adjusting the value of the resistance only, keeping all the other components the same, since $Q = (X_L \text{ or } X_C)/R$.

3.2.7 Bandwidth of a Series RLC Resonance Circuit

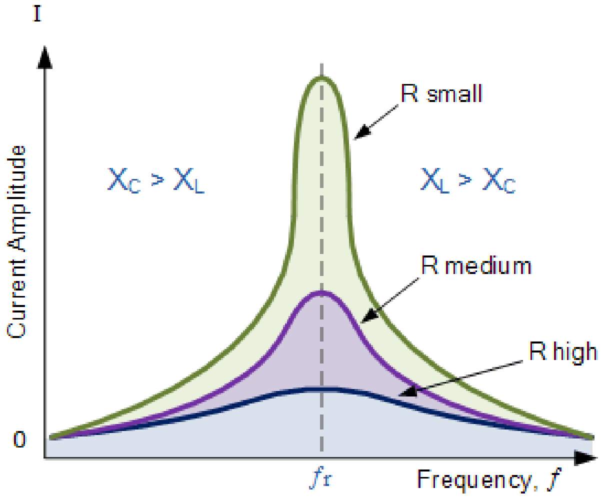


Fig 3.8

Then the relationship between resonance, bandwidth, selectivity and quality factor for a series resonance circuit being defined as:

1). Resonant Frequency, (f_r)

$$X_L = X_C \Rightarrow \omega_r L - \frac{1}{\omega_r C} = 0$$

$$\omega_r^2 = \frac{1}{LC} \quad \therefore \quad \omega_r = \frac{1}{\sqrt{LC}}$$

2). Current, (I)

$$\text{at } \omega_r \quad Z_T = \text{min}, \quad I_S = \text{max}$$

$$I_{max} = \frac{V_{max}}{Z} = \frac{V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_{max}}{\sqrt{R^2 + \left(\omega_r L - \frac{1}{\omega_r C}\right)^2}}$$

3). Lower cut-off frequency, (f_L)

$$\text{At half power, } \frac{P_m}{2},\ I = \frac{I_m}{\sqrt{2}} = 0.707I_m$$

$$Z = \sqrt{2}R,\ X = -R\ (\text{capacitive})$$

$$\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

4). Upper cut-off frequency, (f_H)

$$\text{At half power, } \frac{P_m}{2},\ I = \frac{I_m}{\sqrt{2}} = 0.707I_m$$

$$Z = \sqrt{2}R,\ X = +R\ (\text{inductive})$$

$$\omega_H = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

5). Bandwidth, (BW)

$$BW = \frac{f_r}{Q},\quad f_H - f_L,\quad \frac{R}{L}\ (\text{rads})\ \text{ or }\ \frac{R}{2\pi L}\ (\text{Hz})$$

6). Quality Factor, (Q)

$$Q = \frac{\omega_r L}{R} = \frac{X_L}{R} = \frac{1}{\omega_r CR} = \frac{X_C}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

3.3 Series Resonance Example No1

A series resonance network consisting of a resistor of 30Ω, a capacitor of 2uF and an inductor of 20mH is connected across a sinusoidal supply voltage which has a constant output of 9 volts at all frequencies. Calculate, the resonant frequency, the current at resonance, the voltage across the inductor and capacitor at resonance, the quality factor and the bandwidth of the circuit. Also sketch the corresponding current waveform for all frequencies.

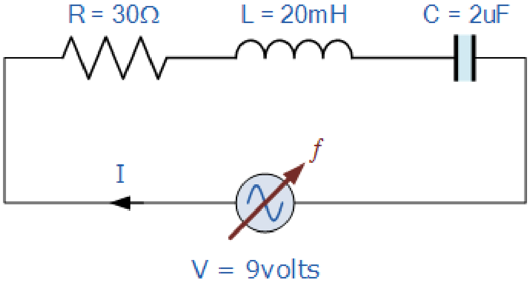


Fig 3.9

Resonant Frequency, f_r

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.02\times 2\times 10^{-6}}} = 796\text{Hz}$$

Circuit Current at Resonance, I_m

$$I = \frac{V}{R} = \frac{9}{30} = 0.3\text{A or } 300\text{mA}$$

Inductive Reactance at Resonance, X_L

$$X_L = 2\pi f L = 2\pi\times 796\times 0.02 = 100\Omega$$

Voltages across the inductor and the capacitor, V_L, V_C

$$V_L = V_C$$
$$V_L = I\times X_L = 300\text{mA}\times 100\Omega$$
$$V_L = 30\text{volts}$$

Note: the supply voltage may be only 9 volts, but at resonance, the reactive voltages across the capacitor, V_C and the inductor, V_L are 30 volts peak!

Quality factor, Q

$$Q = \frac{X_L}{R} = \frac{100}{30} = 3.33$$

Bandwidth, BW

$$BW = \frac{f_r}{Q} = \frac{796}{3.33} = 238\text{Hz}$$

The upper and lower -3dB frequency points, f_H and f_L

$$f_L = f_r - \frac{1}{2}BW = 796 - \frac{1}{2}(238) = 677\text{Hz}$$

$$f_H = f_r + \frac{1}{2}BW = 796 + \frac{1}{2}(238) = 915\text{Hz}$$

Current Waveform

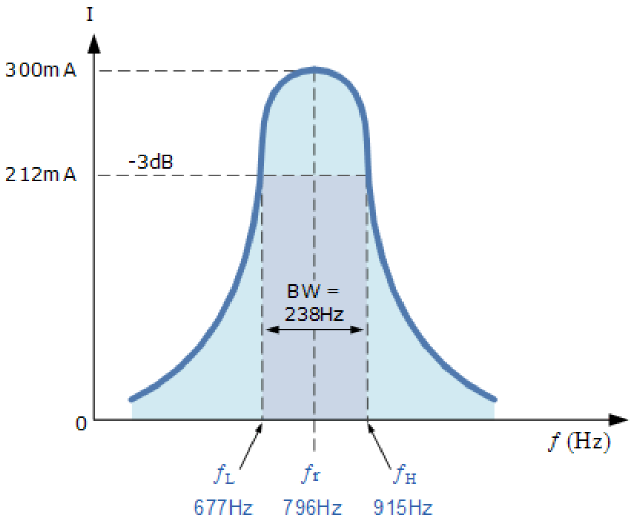


Fig 3.10

3.4 Series Resonance Example No2

A series circuit consists of a resistance of 4Ω, an inductance of 500mH and a variable capacitance connected across a 100V, 50Hz supply. Calculate the capacitance require to give series resonance and the voltages generated across both the inductor and the capacitor.

Resonant Frequency, f_r

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.5 = 157.1\Omega$$

$$\text{at resonance: } X_C = X_L = 157.1\Omega$$

$$\therefore C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi.50.157.1} = 20.3\mu F$$

Voltages across the inductor and the capacitor, V_L, V_C

$$I_S = \frac{V}{R} = \frac{100}{4} = 25\,\text{Amps}$$

$$\text{at resonance: } V_L = V_C$$

$$V_L = I \times X_L = 25 \times 157.1$$
$$\therefore V_L = 3{,}927.5\,\text{volts}$$

Series Resonance Summary

- For resonance to occur in any circuit it must have at least one inductor and one capacitor.
- Resonance is the result of oscillations in a circuit as stored energy is passed from the inductor to the capacitor.
- Resonance occurs when $X_L = X_C$ and the imaginary part of the transfer function is zero.
- At resonance the impedance of the circuit is equal to the resistance value as Z = R.
- At low frequencies the series circuit is capacitive as: $X_C > X_L$, this gives the circuit a leading power factor.
- At high frequencies the series circuit is inductive as: $X_L > X_C$, this gives the circuit a lagging power factor.
- Because impedance is minimum and current is maximum, series resonance circuits are also called **Acceptor Circuits**.

3.5 Parallel Resonance Circuit

In many ways a **parallel resonance** circuit is exactly the same as the series resonance circuit we looked at in the previous tutorial.

Both are 3-element networks that contain two reactive components making them a second-order circuit, both are influenced by variations in the supply frequency and both have a frequency point where their two reactive components cancel each other out influencing the characteristics of the circuit. Both circuits have a resonant frequency point.

Parallel RLC Circuit

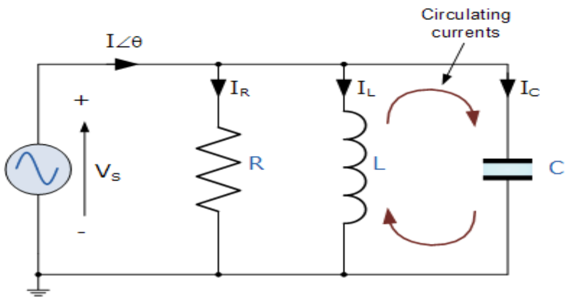


Fig 3.11

A parallel circuit containing a resistance, R, an inductance, L and a capacitance, C will produce a **parallel resonance** (also called anti-resonance) circuit when the resultant current through the parallel combination is in phase with the supply voltage.

In an AC parallel resonance circuits, the supply voltage is common for all branches, so this can be taken as our reference vector. Each parallel branch must be treated separately as with series circuits so that the total supply current taken by the parallel circuit is the vector addition of the individual branch currents.

Resonance occurs when $X_L = X_C$ Then:

$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \ f_r = \frac{1}{2\pi\sqrt{LC}}\ (\text{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}}\ (\text{rads})$$

At resonance the parallel circuit produces the same equation as for the series resonance circuit. Therefore, it makes no difference if the inductor or capacitor is connected in parallel or series. Also at resonance the parallel LC tank circuit acts like an open circuit with the circuit current being determined by the resistor, R only. So the total impedance of a parallel resonance circuit at resonance becomes just the value of the resistance in the circuit and $Z = R$ as shown.

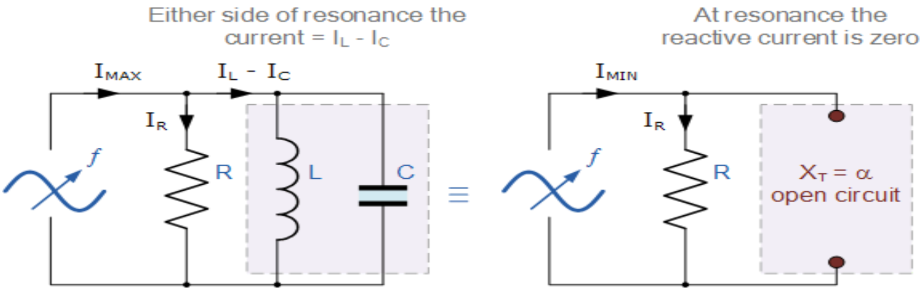


Fig 3.12

At resonance, the impedance of the parallel circuit is at its maximum value and equal to the resistance of the circuit. Also at resonance, as the impedance of the circuit is now that of resistance only, the total circuit current, I will be "in-phase" with the supply voltage, V_S.

We can change the circuit's frequency response by changing the value of this resistance. Changing the value of R affects the amount of current that flows through the circuit at resonance, if both L and C remain constant. Then the impedance of the circuit at resonance $Z = R_{MAX}$ is called the "dynamic impedance" of the circuit.

3.5.1 Impedance in a Parallel Resonance Circuit

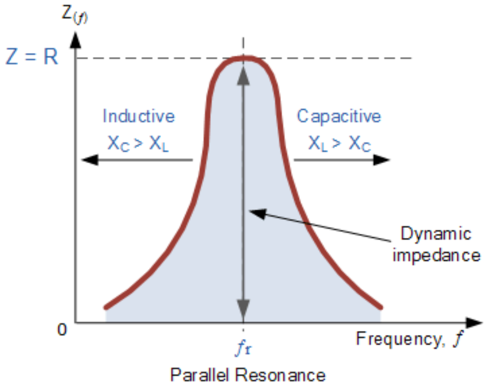


Fig 3.13

Unlike the series resonance circuit, the resistor in a parallel resonance circuit has a damping effect on the circuits bandwidth making the circuit less selective.

Also, since the circuit current is constant for any value of impedance, Z, the voltage across a parallel resonance circuit will have the same shape as the total impedance and for a parallel circuit the voltage waveform is generally taken from across the capacitor.

3.5.2 Current in a Parallel Resonance Circuit

At resonance the current flowing through the circuit must also be at its minimum as the inductive and capacitive branch currents are equal ($I_L = I_C$) and are 180° out of phase.

The total current flowing in a parallel RLC circuit is equal to the vector sum of the individual branch currents and for a given frequency is calculated as:

$$I_R = \frac{V}{R}$$

$$I_L = \frac{V}{X_L} = \frac{V}{2\pi f L}$$

$$I_C = \frac{V}{X_C} = V.2\pi f C$$

$$\text{Therefore,} \quad I_T = \text{vector sum of } (I_R + I_L + I_C)$$

$$I_T = \sqrt{I_R^2 + (I_L + I_C)}$$

At resonance, currents I_L and I_C are equal and cancelling giving a net reactive current equal to zero. Then at resonance the above equation becomes.

$$I_T = \sqrt{I_R^2 + 0^2} = I_R$$

3.5.3 Parallel Circuit Current at Resonance

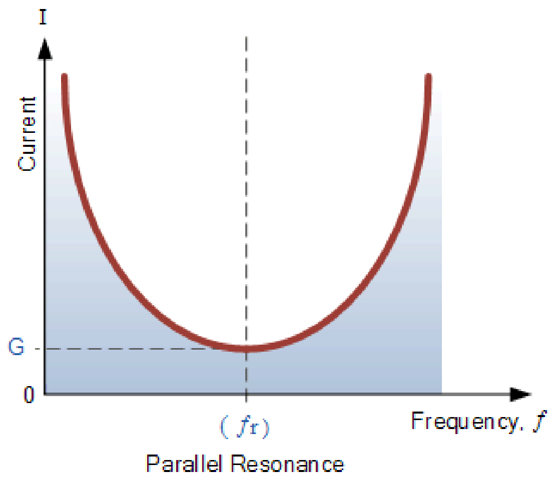


Fig 3.14

The frequency response curve of a parallel resonance circuit shows that the magnitude of the current is a function of frequency and plotting this onto a graph shows us that the response starts at its maximum value, reaches its minimum value at the resonance frequency when $I_{MIN} = I_R$ and then increases again to maximum as f becomes infinite.

The result of this is that the magnitude of the current flowing through the inductor, L and the capacitor, C tank circuit can become many times larger than the supply current, even at resonance but as they are equal and at opposition (180^o out-of-phase) they effectively cancel each other out.

3.5.4 Bandwidth & Selectivity of a Parallel Resonance Circuit

The bandwidth of a parallel resonance circuit is defined in exactly the same way as for the series resonance circuit. The upper and lower cut-off frequencies given as: f_{upper} and f_{lower} respectively denote the half-power frequencies which gives its maximum resonant value, $(0.707 \text{ x } I)^2 R$.

As with the series circuit, if the resonant frequency remains constant, an increase in the quality factor, **Q** will cause a decrease in the bandwidth and likewise, a decrease in the quality factor will cause an increase in the bandwidth as defined by:

$$BW = f_r / Q \quad \text{or} \quad BW = f_{upper} - f_{lower}$$

The selectivity or **Q-factor** for a parallel resonance circuit is generally defined as the ratio of the circulating branch currents to the supply current and is given as:

$$\text{Quality Factor, } Q = \frac{R}{2\pi f L} = 2\pi f CR = R\sqrt{\frac{C}{L}}$$

Note that the Q-factor of a parallel resonance circuit is the inverse of the expression for the Q-factor of the series circuit. Also in series resonance circuits the Q-factor gives the voltage magnification of the circuit, whereas in a parallel circuit it gives the current magnification.

3.5.5 Bandwidth of a Parallel Resonance Circuit

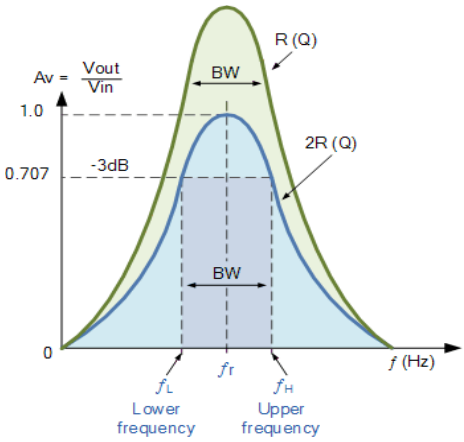
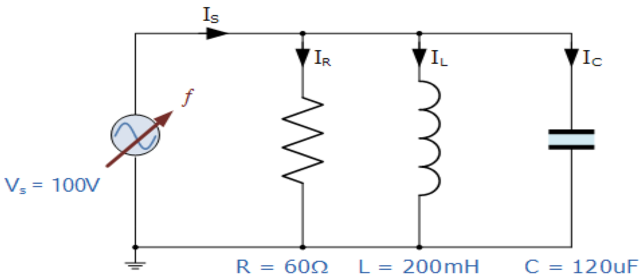


Fig 3.15

Parallel Resonance Example No1

A parallel resonance network consisting of a resistor of 60Ω, a capacitor of 120uF and an inductor of 200mH is connected across a sinusoidal supply voltage which has a constant output of 100 volts at all frequencies. Calculate, the resonant frequency, the quality factor and the bandwidth of the circuit, the circuit current at resonance and current magnification.



Resonant Frequency, f_r

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2.120.10^{-6}}} = 32.5\text{Hz}$$

Inductive Reactance at Resonance, X_L

$$X_L = 2\pi f L = 2\pi.32.5.0.2 = 40.8\Omega$$

Quality factor, Q

$$Q = \frac{R}{X_L} = \frac{R}{2\pi f L} = \frac{60}{40.8} = 1.47$$

Bandwidth, BW

$$BW = \frac{f_r}{Q} = \frac{32.5}{1.47} = 22\text{Hz}$$

The upper and lower -3dB frequency points, f_H and f_L

$$f_L = f_r - \frac{1}{2}BW = 32.5 - \frac{1}{2}(22) = 21.5\text{Hz}$$

$$f_H = f_r + \frac{1}{2}BW = 32.5 + \frac{1}{2}(22) = 43.5\text{Hz}$$

Circuit Current at Resonance, I_T

At resonance the dynamic impedance of the circuit is equal to R

$$I_T = I_R = \frac{V}{R} = \frac{100}{60} = 1.67\text{A}$$

Current Magnification, I_{mag}

$$I_{MAG} = Q \times I_T = 1.47 \times 1.67 = 2.45\text{A}$$

Note that the current drawn from the supply at resonance (the resistive current) is only 1.67 amps, while the current flowing around the LC tank circuit is larger at 2.45 amps. We can check this value by calculating the current flowing through the inductor (or capacitor) at resonance.

$$I_L = \frac{V}{X_L} = \frac{V}{2\pi f L} = \frac{100}{2\pi.32.5.0.2} = 2.45\text{A}$$

Parallel Resonance Summary

We have seen that **Parallel Resonance** circuits are similar to series resonance circuits. Resonance occurs in a parallel RLC circuit when the total circuit current is "in-phase" with the supply voltage as the two reactive components cancel each other out. Also at resonance the current drawn from the supply is also at its minimum and is determined by the value of the parallel resistance.

The equation used to calculate the resonant frequency point is the same for the previous series circuit. However, while the use of either pure or impure components in the series RLC circuit does not affect the calculation of the resonance frequency, but in a parallel RLC circuit it does.

In this tutorial about parallel resonance, we have assumed that the components are purely inductive and purely capacitive with negligible resistance. However in reality the coil will contain some resistance. Then the equation for calculating the parallel resonant frequency of a circuit is therefore modified to account for the additional resistance.

Resonant Frequency using Impure Components

$$f_r = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$$

3.6 Concept of 3 ϕ supply

A three-phase power supply is a special method to alternate power generation, transmission and distribution. This method establishes power supply delivery in three lines, and each line is a phase that carries equal voltage at equal frequency.

There are two types of system available in electric circuit, single phase and **three phase system**. In single phase circuit, there will be only one phase, i.e the current will flow through only one wire and there will be one return path called neutral line to complete the circuit. So in single phase minimum amount of power can be transported. Here the generating station and load station will also be single phase. This is an old system using from previous time.

In 1882, new invention has been done on polyphase system, that more than one phase can be used for generating, transmitting and for load system. **Three phase circuit** is the polyphase system where three phases are send together from the generator to the load. Each phase are having a phase difference of 120°, i.e 120° angle electrically. So from the total of 360°, three phases are equally divided into 120° each. The power in **three phase system** is continuous as all the three phases are involved in generating the total power. The sinusoidal waves for 3 phase system is shown below-

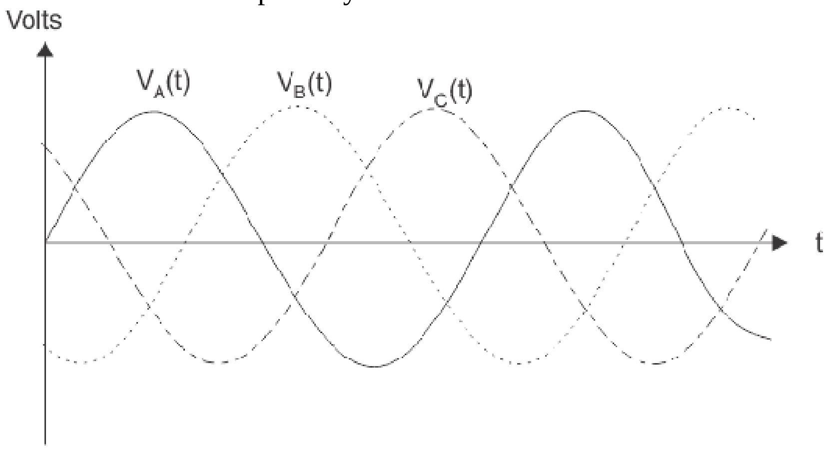


Fig 3.16

The three phases can be used as single phase each. So if the load is single phase, then one phase can be taken from the **three phase circuit** and the neutral can be used as ground to complete the circuit.

In three phase circuit, connections can be given in two types:

1. Star connection
2. Delta connection

3.6.1 Star Connection (Y)

In star connection, there is four wire, three wires are phase wire and fourth is neutral which is taken from the star point. Star connection is preferred for long distance power transmission because it is having the neutral point. In this we need to come to the concept of balanced and unbalanced current in power system.

When equal current will flow through all the three phases, then it is called as balanced current. And when the current will not be equal in any of the phase, then it is unbalanced current. In this case, during balanced condition there will be no current flowing through the neutral line and hence there is no use of the neutral terminal. But when there will be unbalanced current flowing in the three phase circuit, neutral is having a vital role. It will take the unbalanced current through to the ground and protect the transformer. Unbalanced current affects transformer and it may also cause damage to the transformer and for this star connection is preferred for long distance transmission.

In this system of interconnection, the starting ends or finishing ends (Similar ends) of three coils are connected together to form the neutral point. The other ends are joined to the line wires. The common point is called the neutral or Star Point, which is represented by N.

Star Connection is also called **Three Phase 4 wires (3-Phase, 4-Wires)** system.

In star connection, the line voltage is √3 times of phase voltage. Line voltage is the voltage between two phases in three phase circuit and phase voltage is the voltage between one phase to the neutral line. And the current is same for both line and phase. It is shown as expression below

$$E_{Line} = \sqrt{3}E_{phase} \; and \; I_{Line} = I_{Phase}$$

The power in three phase circuit can be calculated from the equation below,

$$P_{Total} = 3 \times E_{phase} \times I_{phase} \times PF$$

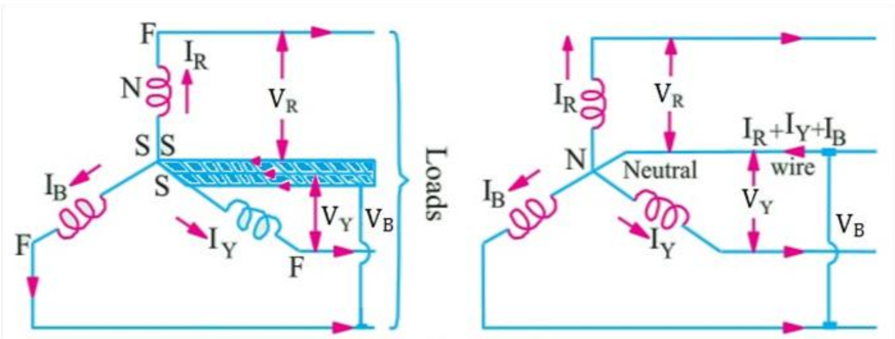


Fig 3.17. Voltage, Current and Power Values in Star Connection (Y)

If a balance symmetrical load is connected across three phase voltage system in parallel, then the three currents will flow in neutral wire which quantities would be same, but they would be differ by 120° (out of phase), hence the vector sum of these three currents = 0.

i.e. $I_R + I_Y + I_B = 0$ ……………. Vectorially

The voltage between any two terminals or Voltage between Line and Neutral (Star Point) is called Phase voltage or Star voltage. And the voltage between two Lines is called Line to Line Voltage or Line Voltage.

1. Line Voltages and Phase Voltages in Star Connection

We know that the Line Voltage between Line 1 and Line 2 is

$V_{RY} = V_R - V_Y$ …. (Vector Difference)

Thus, to find vector of V_{RY}, increase the Vector of V_Y in reverse direction as shown in the dotted form in the below fig . Similarly, on the both ends of vector V_R and Vector V_Y, make perpendicular dotted lines which look like a parallelogram as shown in fig. The Diagonal line which divides the parallelogram into two parts, showing the value of V_{RY}. The angle between V_Y and V_R vectors is 60°.

Hence, if $V_R = V_Y = V_B = V_{PH}$, then

$V_{RY} = 2 \times V_{PH} \times \cos(60°/2)$

$= 2 \times V_{PH} \times \cos 30°$

$= 2 \times V_{PH} \times (\sqrt{3}/2)$ …… Since $\cos 30° = \sqrt{3}/2$

$= \sqrt{3}\ V_{PH}$

Similarly,

$V_{YB} = V_Y - V_B$

$= \sqrt{3}\ V_{PH}$

And

$V_{BR} = V_B - V_R$

$= \sqrt{3}\ V_{PH}$

Hence, it is proved that $V_{RY} = V_{YB} = V_{BR}$ is line voltages V_L in Star Connection,

Therefore, in Star Connection;

$\mathbf{V_L = \sqrt{3}\ V_{PH}}$

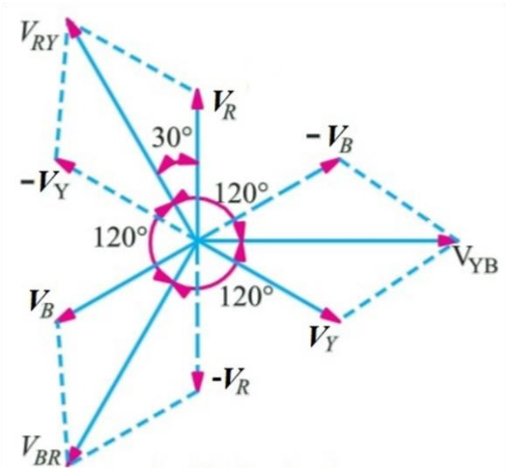


Fig 3.18

It is seen from the fig that;

- Line voltages are 120° apart from each other
- Line voltages are 30° leading from the corresponding phase voltages
- The angle Φ between line currents and respective line voltages are (30°+Φ), i.e. each line current is lagging (30°+Φ) from the corresponding line voltage.

2. Line Currents and Phase Currents in Star Connection

It is seen from the fig (a) that each line is in series with individual phase winding, therefore, the value of line current is same as in Phase windings to which the line is connected. i.e.;

- Current in Line 1 = I_R
- Current in Line 2 = I_Y
- Current in Line 3 = I_B

Since, the flowing currents in all three lines are same, and the individual current in each line is equal to the corresponding phase current, therefore;

$I_R = I_Y = I_B = I_{PH}$ …. The phase current

Line Current = Phase Current

$\mathbf{I_L = I_{PH}}$

In simple words, the value of Line Current and Phase Current is same in Star Connection.

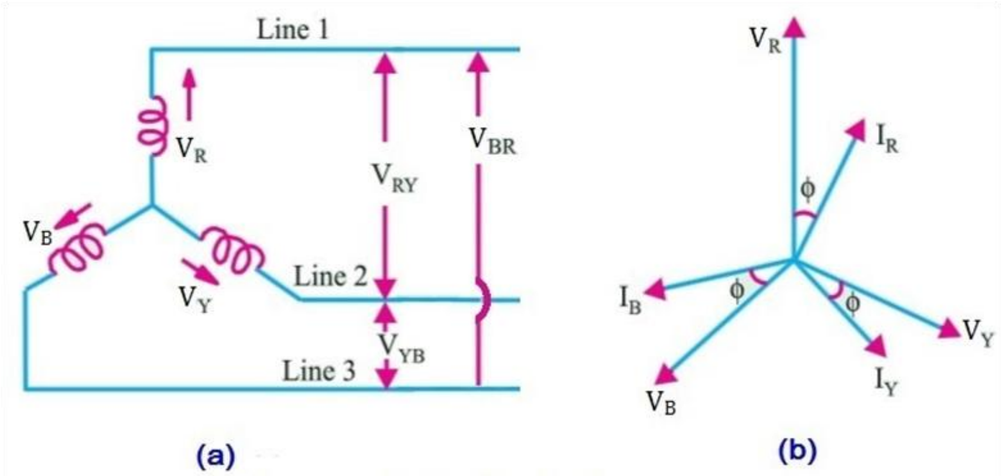


Fig 3.19

3. Power in Star Connection

In a three phase AC circuit, the total True or Active power is the sum of the three phase power.

Hence, total active or true power in a three phase AC system;

$\mathbf{P = 3 \times V_{PH} \times I_{PH} \times Cos\Phi}$ ….. Eq … (1)

Where Cos Φ = Power factor

= the phase angle between Phase Voltage and Phase Current and not between Line current and line voltage.

We know that the values of Phase Current and Phase Voltage in Star Connection;

$\mathbf{I_L = I_{PH}}$

$\mathbf{V_{PH} = V_L/\sqrt{3}}$ ….. (From $V_L = \sqrt{3}\ V_{PH}$)

Putting these values in power eq……. (1)

$P = 3 \times (V_L/\sqrt{3}) \times I_L \times Cos\Phi$ ………. ($V_{PH} = V_L/\sqrt{3}$)

$P = \sqrt{3} \times \sqrt{3} \times (V_L/\sqrt{3}) \times I_L \times Cos\Phi$ ……. {$3 = \sqrt{3}\times\sqrt{3}$}

$P = \sqrt{3} \times V_L \times I_L \times Cos\Phi$

Hence proved;

Power in Star Connection,

$\mathbf{P = 3 \times V_{PH} \times I_{PH} \times Cos\Phi}$ **or**

$\mathbf{P = \sqrt{3} \times V_L \times I_L \times Cos\Phi}$

3.6.2 Delta Connection (Δ)

In delta connection, there is three wires alone and no neutral terminal is taken. Normally delta connection is preferred for short distance due to the problem of unbalanced current in the circuit.

In this system of interconnection, the starting ends of the three phases or coils are connected to the finishing ends of the coil.

In more clear words, all three coils are connected in series to form a close mesh or circuit. Three wires are taken out from three junctions and the all outgoing currents from junction assumed to be positive.

In Delta connection, the three windings interconnection looks like a short circuit, but this is not true, if the system is balanced, then the value of the algebraic sum of all voltages around the mesh is zero.

When a terminal is open, then there is no chance of flowing currents with basic frequency around the closed mesh.

At any instant, the EMF value of one phase is equal to the resultant of the other two phases EMF values but in the opposite direction.

Delta or Mesh Connection System is also called **Three Phase Three Wire System (3-Phase 3 Wire)** and it is the best and suitable system for AC Power Transmission.

In delta connection, the line voltage is same with that of phase voltage. And the line current is √3 times of phase current.

It is shown as expression below,

$$E_{Line} = E_{phase} \text{ and } I_{Line} = \sqrt{3} I_{Phase}$$

The power in three phase circuit can be calculated from the equation below,

$$P_{Total} = 3 \times E_{phase} \times I_{phase} \times PF$$

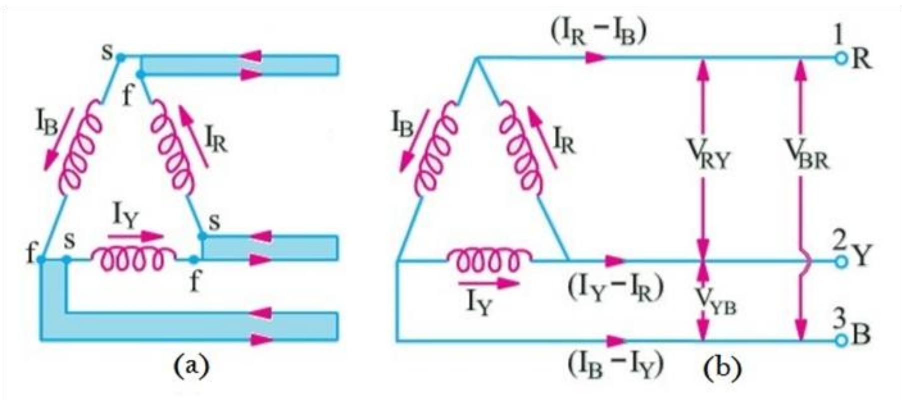


Fig. 3.20 Voltage, Current and Power Values in Delta Connection (Δ)

1. Line Voltages and Phase Voltages in Delta Connection

In Delta Connection, the voltage between (any pair of) two lines is equal to the phase voltage of the phase winding which is connected between two lines. Since the phase sequence is R → Y → B, therefore, the direction of voltage from R phase towards Y phase is positive (+), and the voltage of R phase is leading by 120°from Y phase voltage. Likewise, the voltage of Y phase is leading by 120° from the phase voltage of B and its direction is positive from Y towards B.

If the line voltage between;

Line 1 and Line 2 = V_{RY}

Line 2 and Line 3 = V_{YB}

Line 3 and Line 1 = V_{BR}

Then, we see that V_{RY} leads V_{YB} by 120° and V_{YB} leads V_{BR} by 120°.
Let's suppose,

$V_{RY} = V_{YB} = V_{BR} = V_L$ …………… (Line Voltage)

Then

$\mathbf{V_L = V_{PH}}$

I.e. in Delta connection, the Line Voltage is equal to the Phase Voltage.

2. Line Currents and Phase Currents in Delta Connection

It will be noted from the below fig that the total current of each Line is equal to the vector difference between two phase currents flowing through that line. i.e.;

- Current in Line 1= $I_1 = I_R - I_B$
- Current in Line 2 =$I_2 = I_Y - I_R$
- Current in Line 3 =$I_3 = I_B - I_Y$

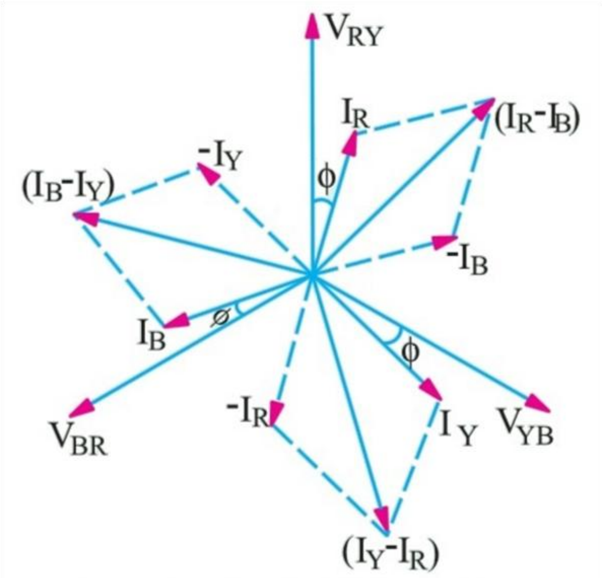


Fig 3.21

The current of Line 1 can be found by determining the vector difference between I_R and I_B and we can do that by increasing the I_B Vector in reverse, so that, I_R and I_B makes a parallelogram. The diagonal of that parallelogram shows the vector difference of I_R and I_B which is equal to Current in Line 1= I_1. Moreover, by reversing the vector of I_B, it may indicate as ($-I_B$), therefore, the angle between I_R and $-I_B$ (I_B, when reversed = $-I_B$) is 60°. If,

$I_R = I_Y = I_B = I_{PH}$ …. The phase currents

Then;

The current flowing in Line 1 would be;

I_L or $I_1 = 2 \times I_{PH} \times \cos(60°/2)$

$= 2 \times I_{PH} \times \cos 30°$

$= 2 \times I_{PH} \times (\sqrt{3}/2)$ …… Since $\cos 30° = \sqrt{3}/2$

$= \sqrt{3}\ I_{PH}$

i.e. In Delta Connection, The Line current is √3 times of Phase Current

Similarly, we can find the reaming two Line currents as same as above. i.e.,

$I_2 = I_Y - I_R$ … Vector Difference = $\sqrt{3}\ I_{PH}$

$I_3 = I_B - I_Y$ … Vector difference = $\sqrt{3}\ I_{PH}$

As, all the Line current are equal in magnitude i.e.

$I_1 = I_2 = I_3 = I_L$
Hence

$IL = \sqrt{3}\ I_{PH}$

It is seen from the fig above that;

- The Line Currents are 120° apart from each other
- Line currents are lagging by 30° from their corresponding Phase Currents
- The angle Φ between line currents and respective line voltages is (30°+Φ), i.e. each line current is lagging by (30°+Φ) from the corresponding line voltage.

3. Power in Delta Connection

We know that the power of each phase

Power / Phase = V_{PH} x I_{PH} x CosΦ

And the total power of three phases;

Total Power = P = 3 x V_{PH} x I_{PH} x CosΦ ….. (1)

We know that the values of Phase Current and Phase Voltage in Delta Connection;

$I_{PH} = I_L / /\sqrt{3}$ ….. (From $I_L = \sqrt{3}\ I_{PH}$)
$V_{PH} = V_L$
Putting these values in power eq……. (1)
P = 3 x V_L x ($I_L/\sqrt{3}$) x CosΦ …… ($I_{PH} = I_L / /\sqrt{3}$)
P = √3 x√3 x V_L x ($I_L/\sqrt{3}$) x CosΦ …{ 3 = √3x√3 }
P = √3 x V_Lx I_L x CosΦ …
Hence proved;

Power in Delta Connection,

P = 3 x V_{PH} x I_{PH} x CosΦ …. or
P = √3 x V_L x I_L x CosΦ

In three phase circuit, star and delta connection can be arranged in four different ways-

1. Star-Star connection
2. Star-Delta connection
3. Delta-Star connection
4. Delta-Delta connection

But the power is independent of the circuit arrangement of the three phase system. The net power in the circuit will be same in both star and delta connection.

The power in three phase circuit can be calculated from the equation below,

$$P_{Total} = 3 \times E_{phase} \times I_{phase} \times PF$$

Since, there is three phases, so the multiple of 3 is made in the normal power equation and the PF is power factor. Power factor is a very important factor in three phase system and some times due to certain error, it is corrected by using capacitors.

3.6.3 Difference between Star & Delta.

Star (Y) Connection	**Delta (Δ) Connection**
In STAR connection, the starting or finishing ends (Similar ends) of three coils are connected together to form the neutral point. A common wire is taken out from the neutral point which is called Neutral.	In DELTA connection, the opposite ends of three coils are connected together. In other words, the end of each coil is connected with the start of another coil, and three wires are taken out from the coil joints
There is a **Neutral** or **Star Point**	No Neutral Point in Delta Connection
Three phase four wire system is derived from Star Connections **(3-Phase, 4 Wires System**) We may Also derived 3 Phase 3 Wire System from Star Connection	Three phase three wire system is derived from Delta Connections **(3-Phase, 3 Wires System)**
Line Current is Equal to Phase Current. i.e. Line Current = Phase Current $\mathbf{I_L = I_{PH}}$	Line Voltage is Equal to Phase Voltage. i.e. Line Voltage = Phase Voltage $\mathbf{V_L = V_{PH}}$
Line Voltage is √3 times of Phase Voltage. i.e. $\mathbf{V_L = \sqrt{3}\ V_{PH}}$	Line Current is √3 times of Phase Current. i.e. $\mathbf{I_L = \sqrt{3}\ I_{PH}}$
The Total Power of three phases could be found by $\mathbf{P = \sqrt{3} \times V_L \times I_L \times Cos\Phi}$ **…. Or** $\mathbf{P = 3 \times V_{PH} \times I_{PH} \times Cos\Phi}$	The Total Power of three phases could be found by $\mathbf{P = \sqrt{3} \times V_L \times I_L \times Cos\Phi}$ **… or** $\mathbf{P = 3 \times V_{PH} \times I_{PH} \times Cos\Phi}$
The speeds of Star connected motors are slow as they receive **1/√3** voltage.	The speeds of Delta connected motors are high because each phase gets the total of line voltage
In Star Connection, the phase voltage is low as 1/√3 of the line voltage, so, it needs low number of turns, hence, saving in copper.	In Delta connection, The phase voltage is equal to the line voltage, hence, it needs more number of turns.
Low insulation required as phase voltage is low	Heavy insulation required as Phase voltage = Line Voltage.
In Power Transmission, Star Connection system is general and typical to be used.	In Power Distribution and industries, Delta Connection is general and typical to be used.

3.7 Methods of measurements of Three phase power

Various methods are used for **measurement of three phase power** in three phase circuits on the basis of number of wattmeter used. We have three methods

1. Three wattmeters method
2. Two wattmeters method
3. Single wattmeter method.

Measurement of Three Phase Power by Two Wattmeters Method

In this method we have two types of connections

1. Star connection of loads
2. Delta connection of loads.

When the star connected load, the diagram is shown in below-

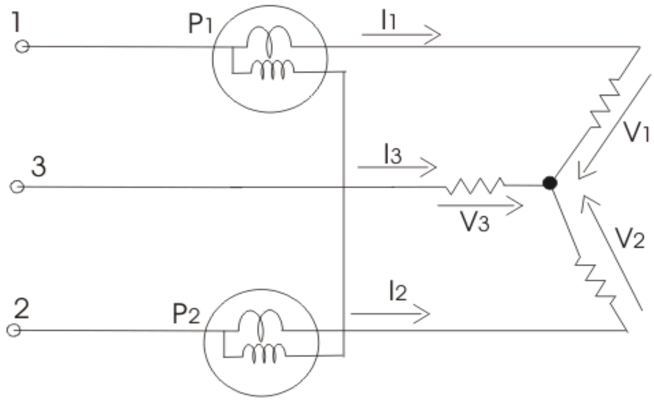


Fig 3.22

For star connected load clearly the reading of wattmeter one is product phase current and voltage difference (V_2-V_3). Similarly the reading of wattmeter two is the product of phase current and the voltage difference (V_2-V_3). Thus the total power of the circuit is sum of the reading of both the wattmeters. Mathematically we can write

$$P = P_1 + + P_2 = I_1(V_1 + V_2) + I_2(V_2 - V_3)$$

but we have $I_1+I_2+I_3=0$,hence putting the value of $I_1+I_2= -I_3$.

We get total power as $V_1I_1+V_2I_2+V_3I_3$.

When delta connected load, the diagram is shown in below

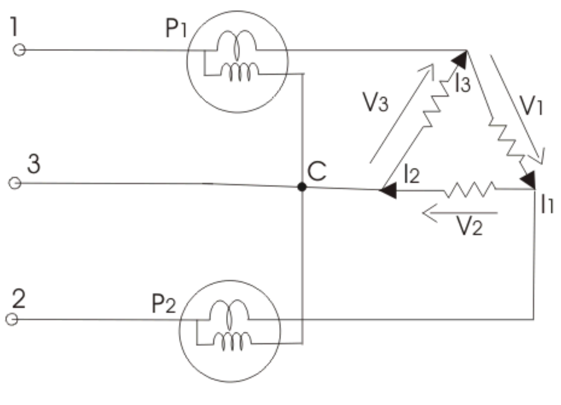


Fig 3.23

The reading of wattmeter one can be written as

$$P_1 = -V_3(I_1 - I_3)$$

and reading of wattmeter two is

$$P_2 = -V_2(I_2 - I_1)$$

$$Total\ power\ is\ \ P = P_1 + P_2 = V_2I_2 + V_3I_3 - I_1(V_2 + V_3)$$

but $V_1+V_2+V_3=0$,

hence expression for total power will reduce to $V_1I1+V_2I_2+V_3I_3$.

3.8 Advantages of Three Phase Over Single Phase

Three phase system is widely used in generation, transmission and distribution. Single phase system is only used to operate small electrical appliances at consumer ends where the power rating is very less. Some of the advantages of three phase system or poly phase systems compared to single phase system is explained below

Advantages of three phase system:

- **The output of 3 phase machine is always greater than single phase machine of same size.** The output will be approximately 1.5 times than single phase machine. So for given size and voltage 3 phase alternator or electrical machines occupy less space and less cost compared to single phase machine having same rating
- **For transmission of electrical power three phase supply requires less copper or less conducting material than that of single phase system** for given volt-amperes and voltage ratings. Hence 3 phase system is more economical compared to single phase system
- **Single phase machines are not self starting machines. On the other hand three phase machines are self starting due to rotating magnetic field**. Therefore in order to start a single phase machine an auxiliary device is required which not in the case of 3 phase machine.
- **Power factor of single phase machines is poor compared to three phase machines.**
- **In single phase system the instantaneous power is function of time. Hence fluctuates with respect to time.** The fluctuating power will cause significant vibrations in the single phase machines. Hence performance of single phase machines is poor. **While instantaneous symmetrical three phase system is always constant**
- **Three phase system gives steady output**
- **Single phase system can be obtained from three phase supply system, vice-versa is not possible**
- **For converting systems like rectifiers, the dc voltage waveform becomes more smoother with the increase in the number of phases of the system. Hence three phase system is advantageous compared to single phase system**
- **3 phase motors will have uniform torque whereas single phase motors will have pulsating torque**
- **Parallel operation of three phase generators will be simple compared to single phase generators because of pulsating reaction in single phase generator**

3.9 Simple Problems

Exercise 1: Three loads, each of resistance 50 Ω are connected in star to a 400 V, 3-phase supply. Determine a) the phase voltage, (b) the phase current and (c) the line current.

400 V, 3-phase supply means that 400 V is the line voltage.

(a) For a star connection, $V_L = \sqrt{3}\,V_P$

Hence, **phase voltage**, $V_P = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}}$ = **231 V**

(b) **Phase current**, $I_P = \dfrac{V_P}{R_P} = \dfrac{231}{50}$ = **4.62 A**

(c) For a star connection, $I_P = I_L$

Hence, **line current**, I_L = **4.62 A**

Exercise 2: A star-connected load consists of three identical coils, each of inductance 159.2 mH and resistance50 Ω. If the supply frequency is 50 Hz and the line current is 3 A determine (a) the phase voltage and (b) the line voltage.

Inductive reactance, $X_L = 2\pi fL = 2\pi(50)(159.2 \times 10^{-3}) = 50\ \Omega$

Impedance of each phase, $Z_p = \sqrt{R^2 + X_L^2} = \sqrt{50^2 + 50^2} = 70.71\ \Omega$

For a star connection, $I_L = I_p = \dfrac{V_p}{Z_p}$

Hence, **phase voltage,** $\mathbf{V_p} = I_p Z_p = (3)(70.71) =$ **212 V**

Line voltage, $\mathbf{V_L} = \sqrt{3}\,V_p = \sqrt{3}\,(212) =$ **367 V**

Exercise 3: Three loads, each of resistance 50 Ω are connected in delta to a 400 V, 3-phase supply. Determine (a) the phase voltage, (b) the phase current and (c) the line current.

(a) For a delta connection, $V_L = V_P$

Since V_L = 400 V, then **phase voltage**, V_P = **400 V**

(b) **Phase current**, $I_P = \dfrac{V_P}{R_P} = \dfrac{400}{50}$ = **8 A**

(c) For a delta connection, **line current**, $I_L = \sqrt{3}\,I_P = \sqrt{3}\,(8)$ = **13.86 A**

UNIT - III

Review Questions

PART – A (2 marks Questions)

1. Define Amplitude.
2. Define Peak value
3. Define Peak to Peak value.
4. Define Cycle.
5. Define Impedance.
6. Define Power factor.
7. Define Quality factor.
8. Define Bandwidth.
9. What is the condition for Resonance ?
10. What is meant by Resonance ?
11. What are the applications of Resonance ?

PART – B (Three marks Questions)

1. Derive the Q-factor of series resonant circuit ?
2. Define and derive the Q-factor of parallel resonant circuit ?
3. Define and derive the bandwidth of series resonant circuit ?
4. Write the condition for series resonance and Explain.
5. Write the condition for parallel resonance and Explain.
6. Derive an expression for frequency at resonance.
7. Write concept of 3ϕ supply.
8. Write the value of line and phase voltage and current in star connected circuit.
9. Write the value of line and phase voltage and current in star connected circuit.
10. Write ant 3 advantages of three phase power over single phase power .

PART – C (Ten Marks Questions)

1. Derive an expression for resonant frequency , Q factor & Bandwidth of a series resonant circuit.
2. Derive an expression for resonant frequency , Q factor & Bandwidth of a parallel resonant circuit.
3. Explain line and phase voltage and current in Star connected circuits.
4. Explain line and phase voltage and current in Delta connected circuits.
5. State relation between phase current and line current, phase voltage and line voltage for following systems: a. Star connected system b. Delta connected system.
6. Expalin measurement of three phase power by two wattmeter method with neat diagram.
7. Write advantages of three phase power over single phase power .

UNIT IV

DC MACHINES AND A.C MACHINES

4.1 INTRODUCTION:

An electrical generator is a machine which converts mechanical energy into electrical energy.

The energy conversion is based on the principle of the production of dynamically induced emf.

D.C. Generator works on the principle of Faraday's law of electromagnetic induction. Whenever a conductor cuts magnetic flux, dynamically induced emf is produced in it according to Faraday's laws. This emf causes a current to flow if the conductor circuit is closed.

4.2 TYPES :

Generators are usually classified according to the way in which their fields are excited. Depending on the method of excitation DC generators are classified as:.

1. Separately excited D.C Generators.
2. Self excited DC Generators.

SEPARATELY EXCITED DC GENERATORS :-

These types of generators are those whose field magnets are energized from an independent external source of D.C current, as shown in fig.4.1

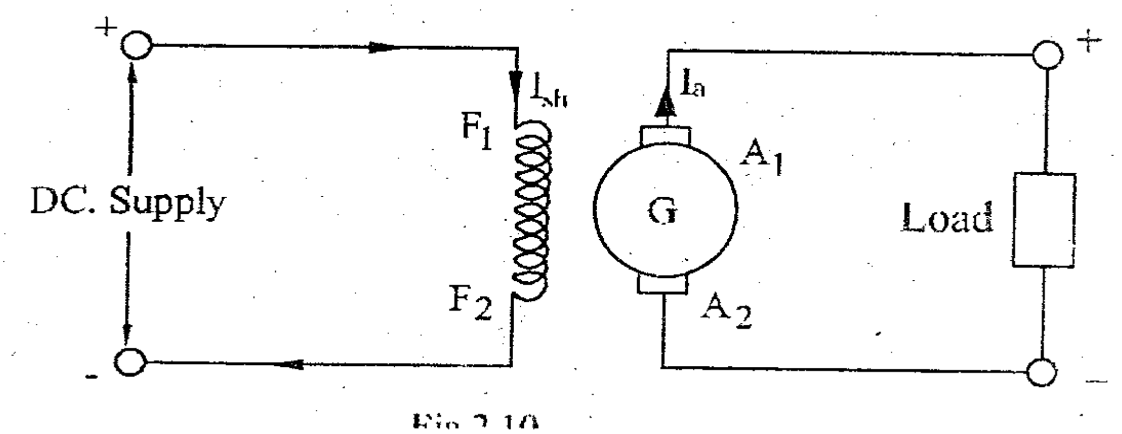


Fig.4.1 Separately excited DC generators

SELF EXCITED D.C GENERATOR:

These types of generator are those whose field magnets are energized by the current produced by the generator themselves. Due to residual magnetism, there is always present some flux in the pole. When the armature is rotated, some emf and hence some induced current is produced. Which is partly or fully passed through the field coils thereby strengthening the residual pole flux.

There are three types of self-excited generators named according to the manner in which their field coils (or windings) are connected to the armature.

1. DC Shunt Generator
2. DC Series Generator
3. DC Compound Generator

DC Shunt Generator:-

In shunt generator the field windings are Connected across or in parallel with the armature conductors as shown in fig.4.2

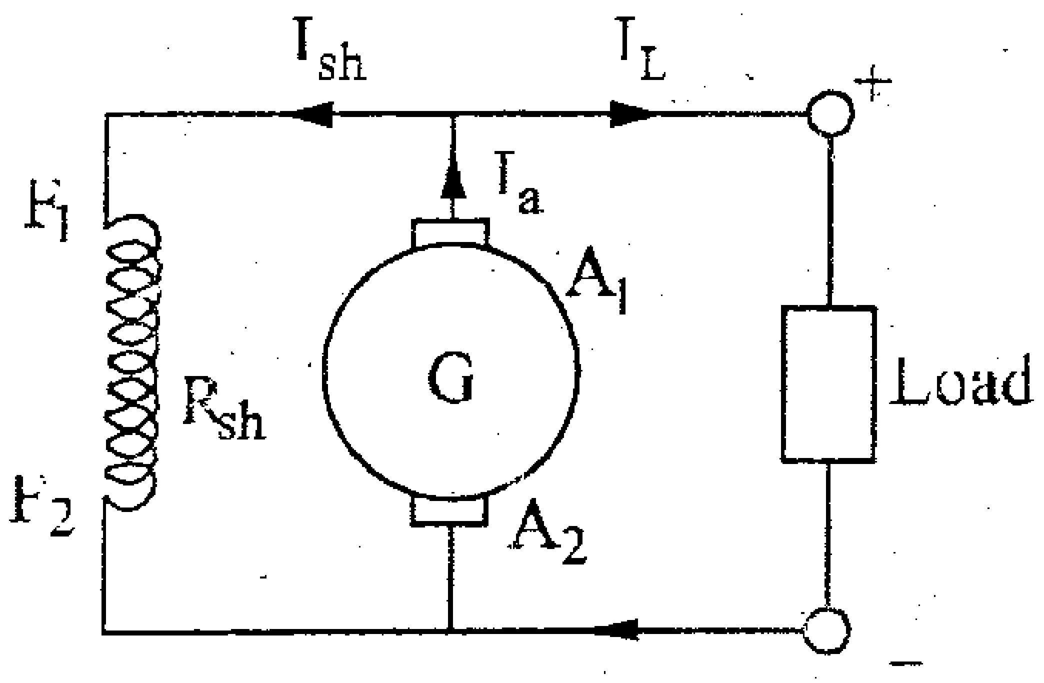


Fig.4.2 DC Shunt Generator

In DC shunt generator the armature current (I_a) will be the sum of load current (I_L) and the field current (I_{sh}).

i.e.

$I_a = I_L + I_{sh}$, $I_{sh} = V/R_{sh}$

I_{sh}- Shunt field current

DC Series Generator:

In D.C Series generator the field windings are Joined in Series with the arrnature conductors, as shown in fig.4.3

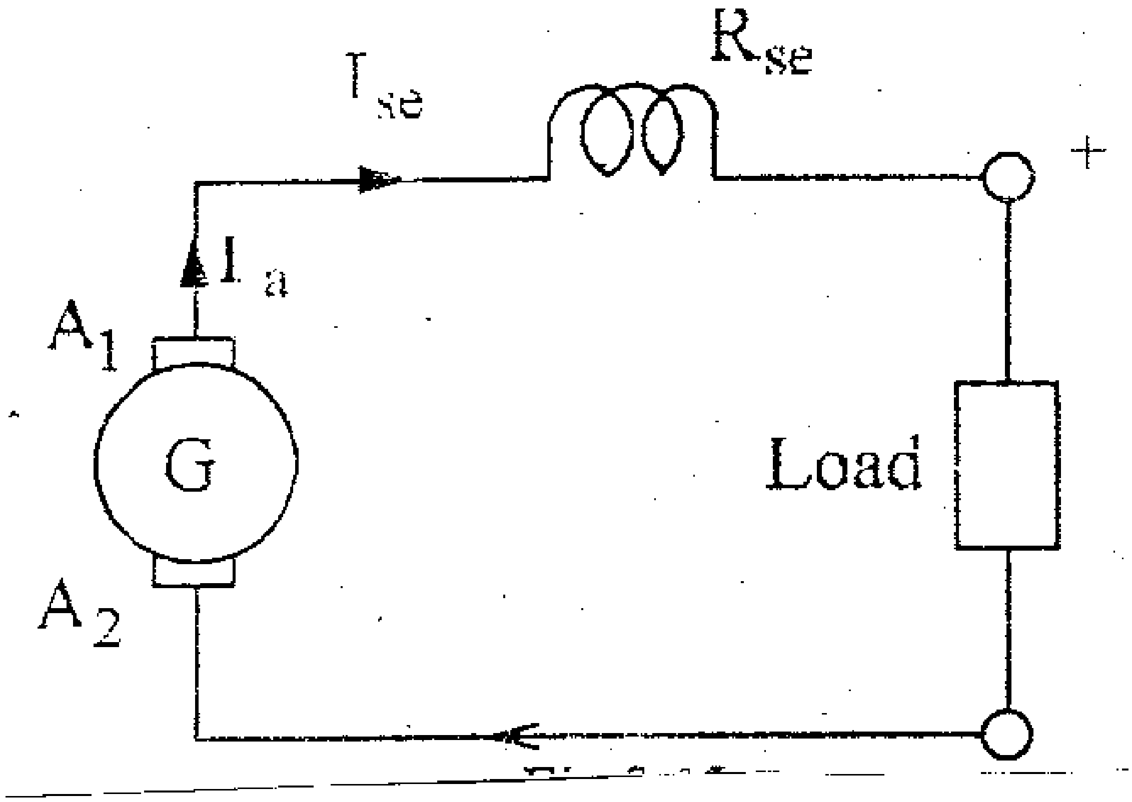


Fig.4.3 Dc Series Generator

DC Series Generator the armature current (Ia), the field current (Ise) and load current (IL) are same.

i.e., $I_a = I_{se} = I_L$

DC Compound Generators :

It is a combination of a few series and a few shunt windings and can be either short-shunt or long-shunt as shown in fig.

1. Long-Shunt DC Generators
2. Short-Shunt DC Generators

In compound generator be the shunt field is stronger than the series field. When series field aids the shunt field generator is said to be commutatively-compound. On the other hand in series field opposes the shunt field, the generator is said to be differentially compounded.

Long Shunt Generator   Short – Shunt Generator

In this generator

$I_a = I_{se} = I_L + I_{sh}$

In this generator

$I_{se} = I_L$

$I_{se} = I_L + I_{sh}$

$I_a = I_{se} + I_{sh}$

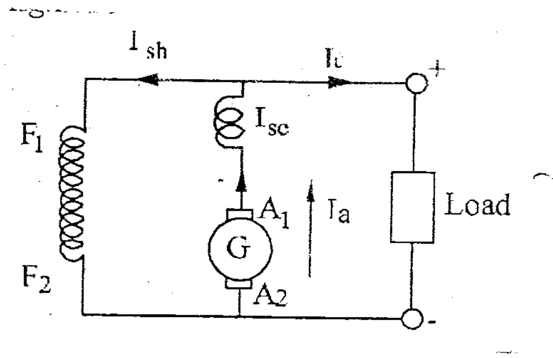


Fig.4.4 Long shunt

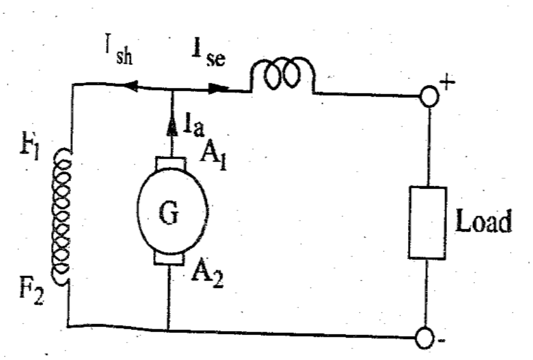


Fig.4.5 Short shunt

4.3 VARIOUS TYPES OF DC GENERATORS

It is shown in fig.4.6

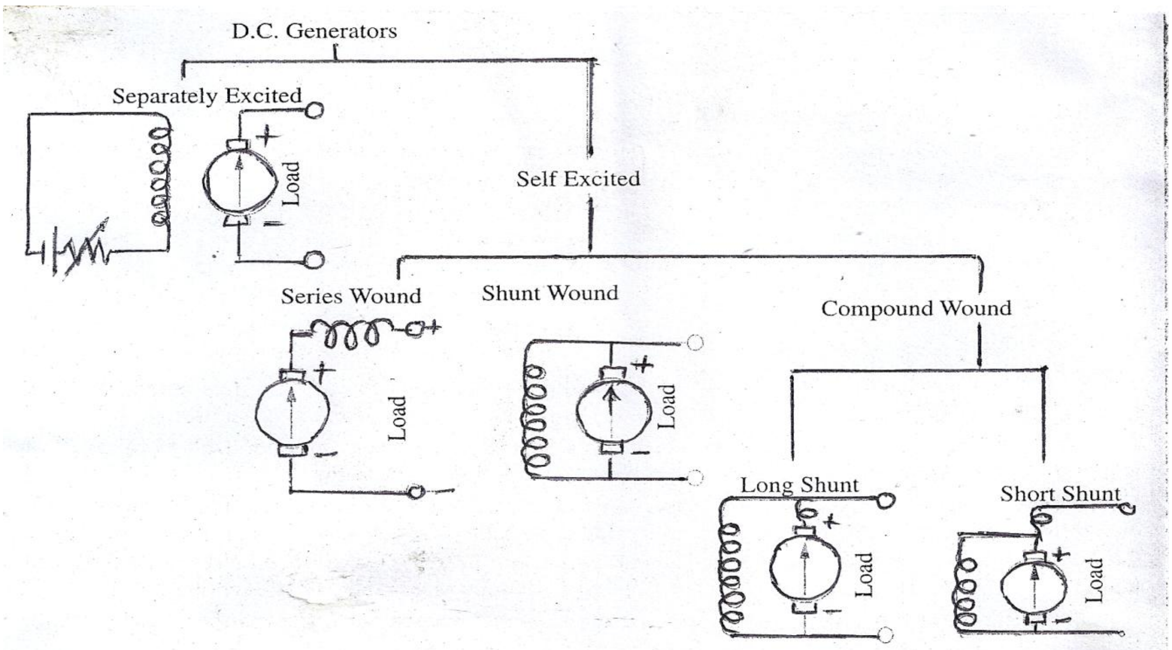


Fig. 4.6 Various Type

4.4 CONSTRUCTION OF D.C. GENERATOR

The Construction of DC Generator is shown in fig .4.7

The DC Generator consists of the following essential parts.

1. Magnetic Frame (or) Yoke.
2. Pole-Cores and Pole Shoes.
3. Pole coils (or) Field Coils
4. Armature Core.
5. Armature Windings (or) Conductors.
6. Commutator.
7. Brushes and Bearings.

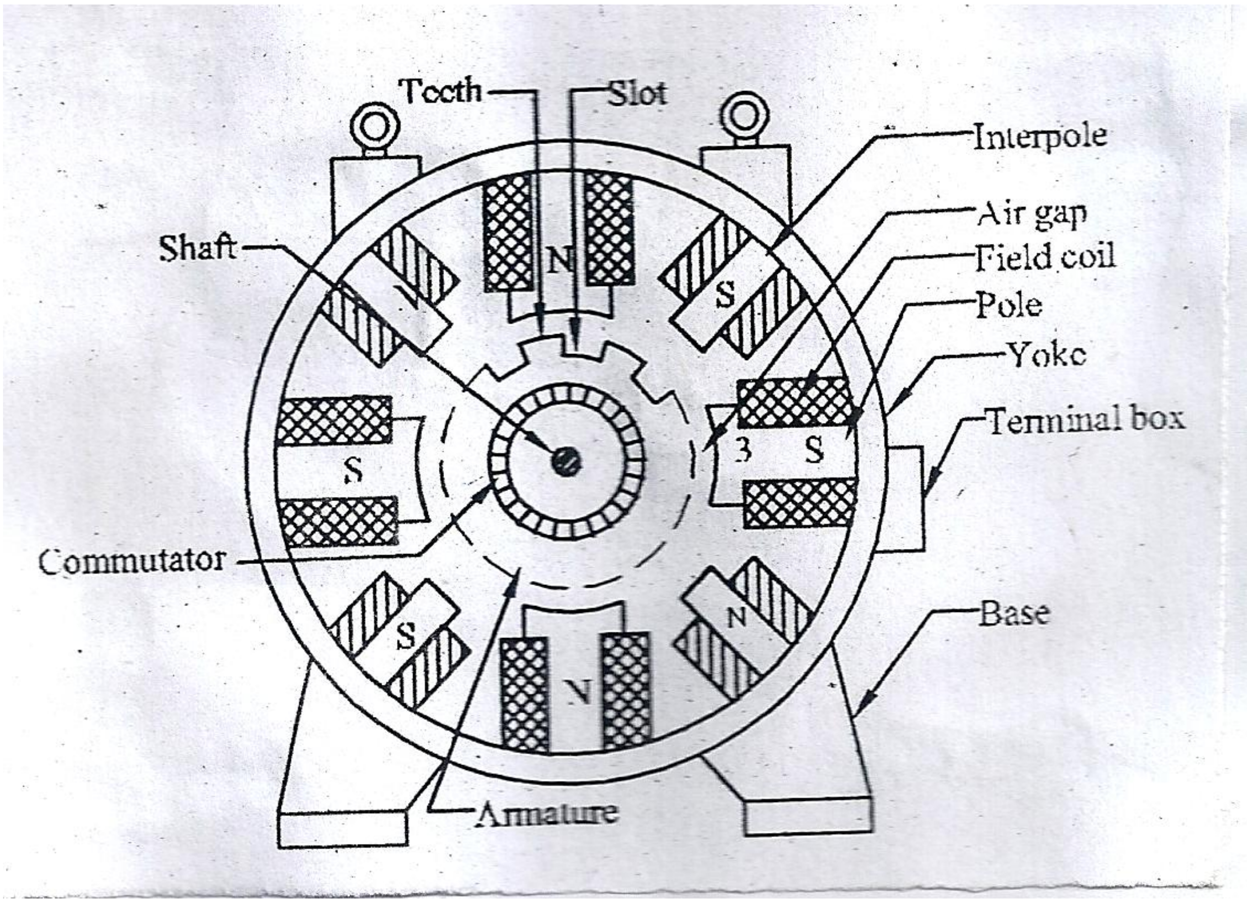


Fig.4.7 DC Machines

1. YOKE (OR) MAGNETIC FRAME :

Fig.4.6 Shows the Yoke. Yoke of a DC generator has two function.

(i) It provides Mechanical support for the poles and acts as a protecting cover for the whole machine.

(ii) It carries the magnetic flux produced by the poles.

In smaller generators the poles and yokes are forged into a single piece, and larger generators poles are made separately and fitted with the inner periphery of the yoke.

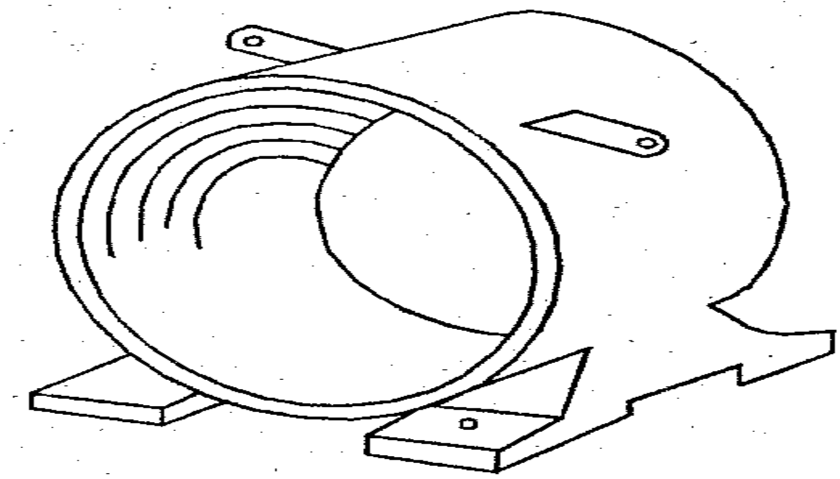


Fig.4.8 YOKE

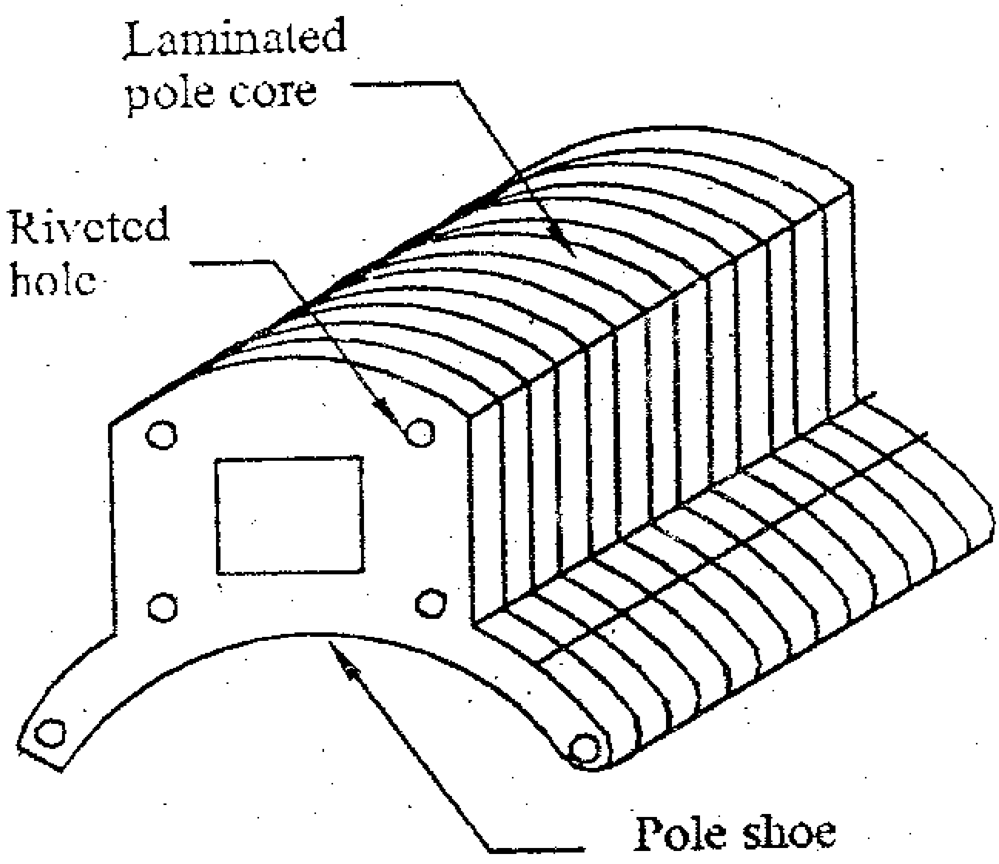


Fig.4.9

2. POLE CORES AND POLE SHOES.

The field magnets consists of pole core and pole shoes. The Shoes serve two purpose.

(i) They spread out the flux in the air gap and also, being of larger cross-section, reduce the reluctance of the magnetic path.

(ii) They support the exciting coils.

There are two main type of pole construction.

(i) The pole core itself may be a solid piece made out of either cast iron (or) cast steel but the pole shoe is laminated and is fastened to the pole face by means of counter sunk screws.

(ii) In modern design, the complete pole core and pole shoes are built of thin lamination of annealed steel which are riveted together under hydraulic pressure. The Thickness of lamination varies from 1 mm to 0.25 mm.

3. POLE COILS (OR) FIELD COILS

The field coils or pole coils, which consists of copper wire or strip, are former wound for the correct dimension. Than the former is removed and wound coil is put into place over the core.

When current is passed through these coils, they electro magnetize the poles which produce the necessary flux that is cut by revolving armature conductors.

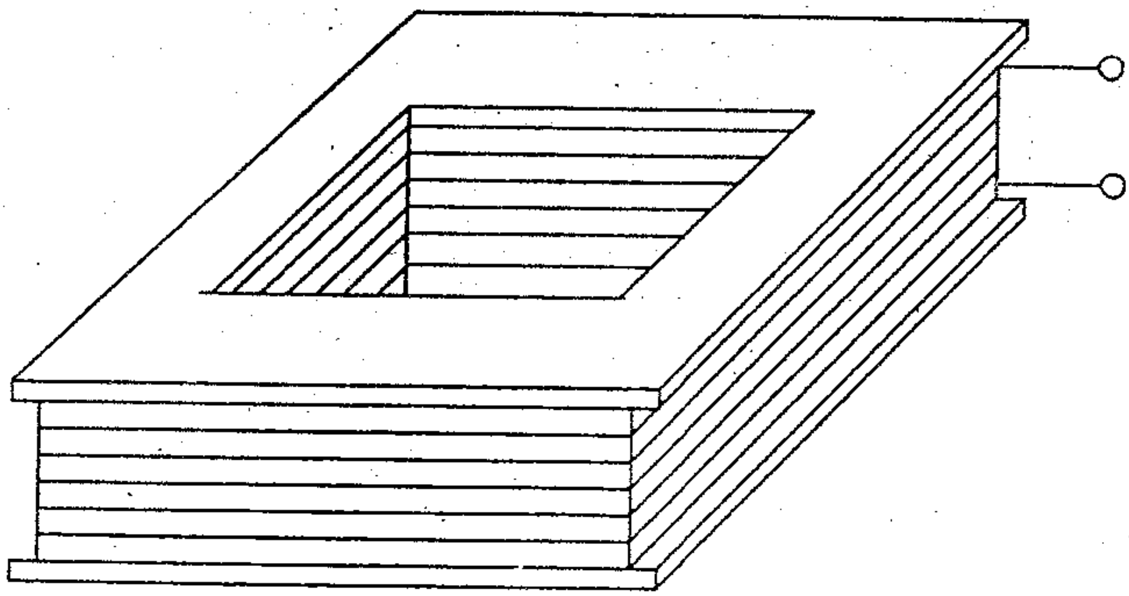


Fig:4.10

4. ARMATURE CORE :

The Armature core is keyed to the machine shaft and it rotates between the field poles. It consists of slotted steel lamination about (0.4 to 0.6 mm) Thickness. These laminations are stacked to form a cylindrical core as shown in fig.

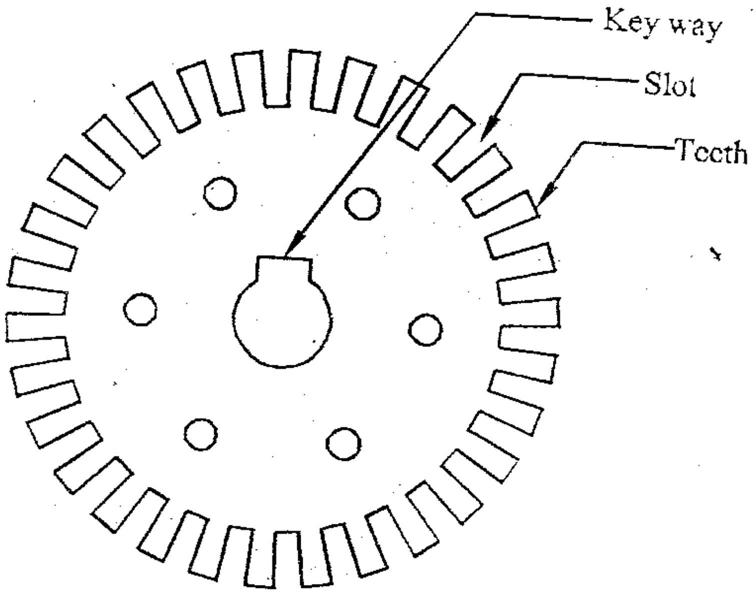


Fig. 4.11

The lamination are insulated from each other by thin coating of varnish. The purpose of laminating the core is to reduce the eddy current loss. The lamination are slotted to accommodate the armature winding. The slots are closed by fibre (or) wooden wedges to prevent the conductors from plying out due to the centrifugal force.

5. ARMATURE WINDINGS (OR) CONDUCTORS:

The Armature windings are usually former-wound these are first wound in the form of flat rectangular coils and are then pulled into their proper shape in a coil puller. Various conductors of the coils are insulated from each other. The conductors are placed in the armature slots which are lined with tough insulating material. Armature winding is wound in two ways.

1. Lap Winding : for low voltage high current Machine
2. Wave Winding : for high voltage low current Machine

6. COMMUTATOR

The function of the commutator is to facilitate collection of current from the armature conductor as shown in fig. It rectified i.e., converts the alternating current in the armature conductors into unidirectional current in the external load circuit. It is cylindrical structure and is built up of wedge shaped segments of high conductivity hard-drawn (or) drop forged copper. These segments are insulated from each other by thin layers of mica. The number coils. Each commutator segment is connected to the armature conductor by means of a copper 1 kg on strip (or) riser.

Commutator

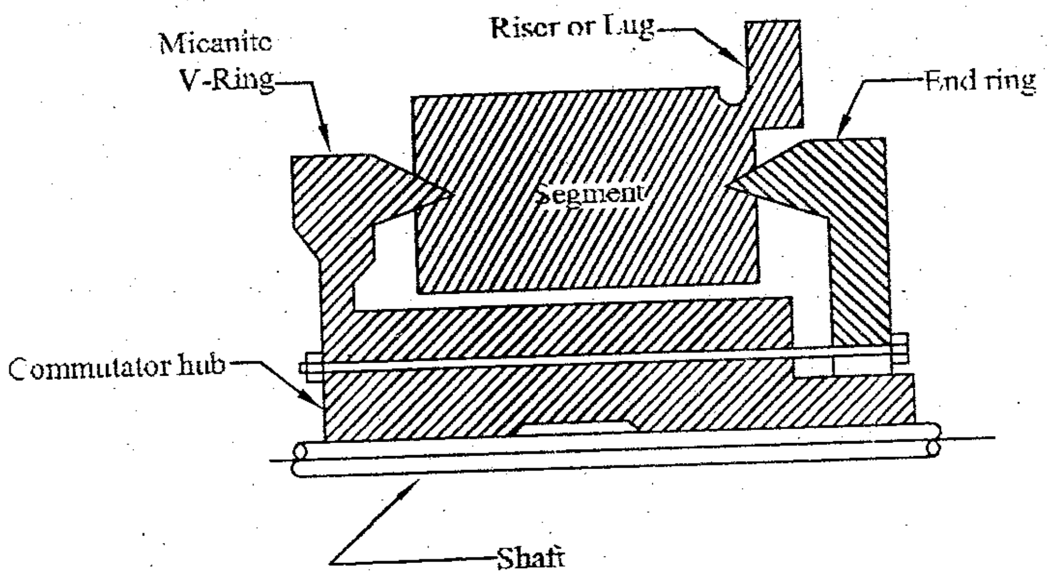


Fig. 4.12

7. BRUSHES AND BEARINGS :

The function of brushes is to collect current from commutator, are usually made of carbon or graphite and are in the shape of a rectangular block. The brushes are put inside the brush holders. The brush holders are kept pressed against the commutator by a spring as shown in fig.

Ball bearing (or) roller bearings are fitted inside the end corer. Armature shaft is mounted over these bearings. Because of their reliability, ball bearings are frequently employed, through for heavy duties roller bearings are preferable.

4.5 PRINCIPLE OF OPERATION OF D.C GENERATOR

A generator is a machine which converts Mechanical energy into electrical energy. The energy conversion is based on the principle of production of dynamically induced emf. Whenever a conductor cuts the magnetic flux, dynamically emf is induced in it, according to Faraday's law of Electromagnetic induction. This emf causes a current to flow, of the conductor circuit is closed. The direction of moved emf is given by "Flemings Right hand Rule". Therefore the important components of a generator are.

(i) A Magnetic field
(ii) Conductor (or) conductors which can so move as to cut the flux.

In DC generators, a stationary magnetic field is produced by field magnet. The armature consisting of conductors is rotated inside the magnetic field by a prime mover. The prime mover may be a turbine (or) diesel (or) Petrol engine. The nature of emf induced in the Armature is AC. The AC emf is converted into DC by means of commutator. The commutator is rotated along with the Armature.

The method of producing emf in a simple loop generator is explained with the help of fig.4.11

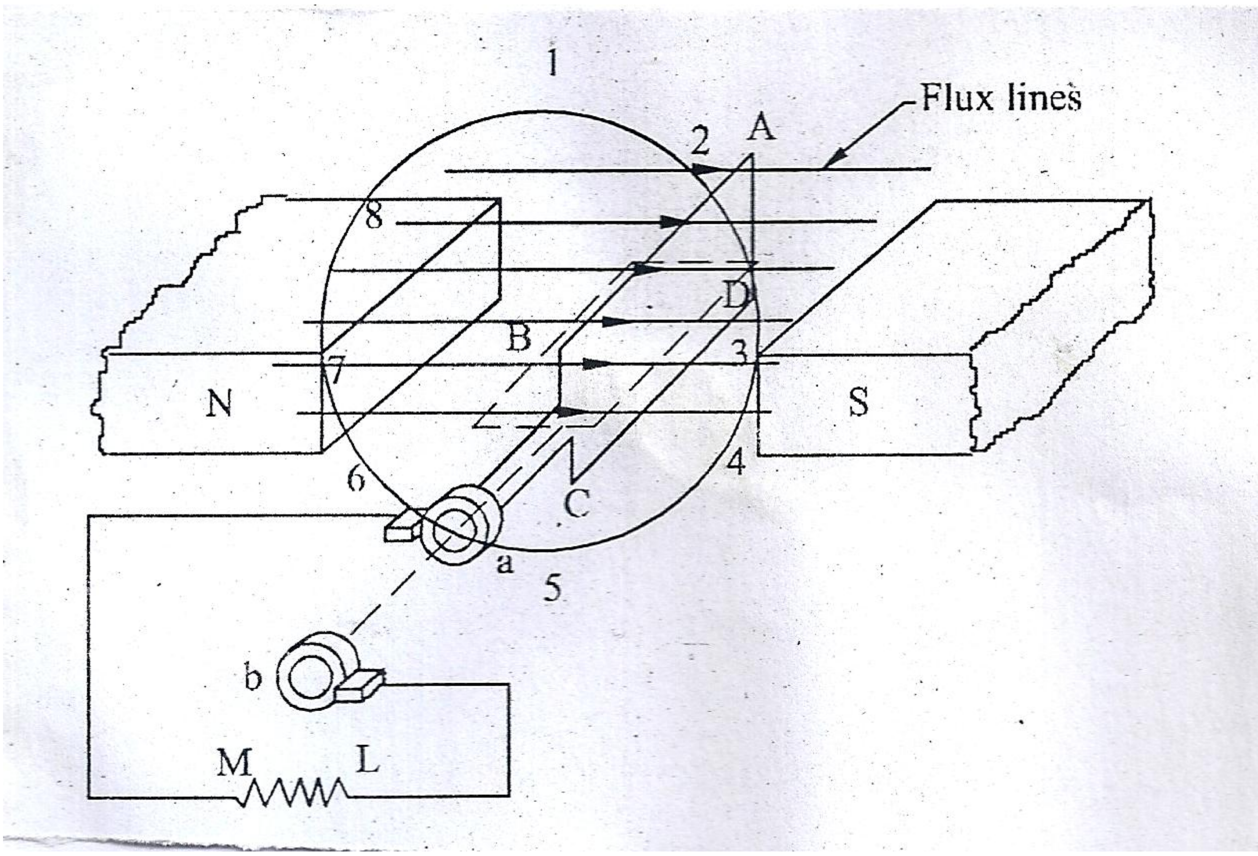


Fig. 4.13

Construction, In the above fig shown a single turn rectangular copper coil ABCD rotating about its own axis in a magnetic field provided by either permanent magnet (or) electromagnets. The two ends of the coil are joined to two slip rings 'a' and 'b' which are insulated from each other and from the central shaft. Two collecting brushes press against the slip rings. Their function is to collect the current induced in the coil and to convey the external load resistance R. The rotating coil may be called "armature" and the magnets as "Field Magnets"

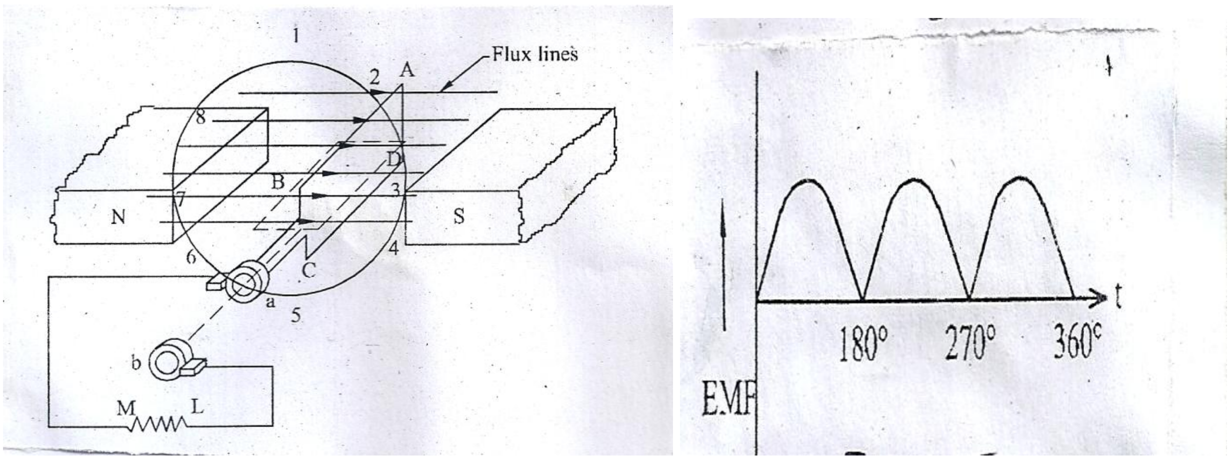


Fig.4.14

In the coil is in the reversed direction. Therefore at position 7, the emf induced is Negative Maximum. Than the coil moves from position 7 to position 1, (270` to 360`) the flux linked with the coil gradually increases, but the rate of change of flux linkages decreases. The emf induced is Zero at position 1. This the emf induced in to the coil is an alternating emf as shown in fig.

If the slip rings are replaced by split rings. The alternating emf will become unidirectional current. The split rings are made out of a conducting cylinder. which is cut into two segments insulated from each other by thin sheet of mica. The coil ends are joined to these segments, carbon brushes rest on the segments.

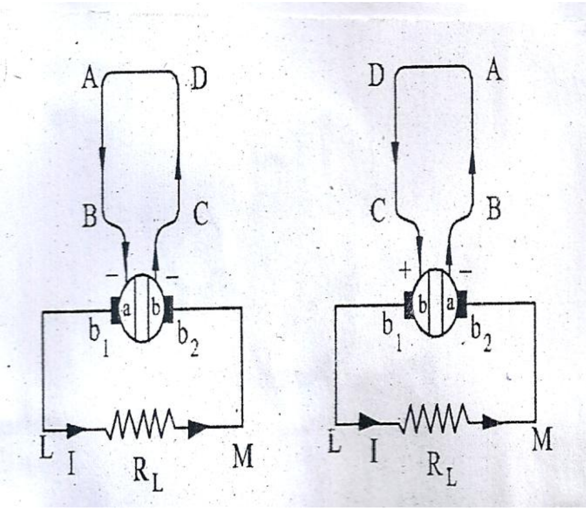


Fig.4:15

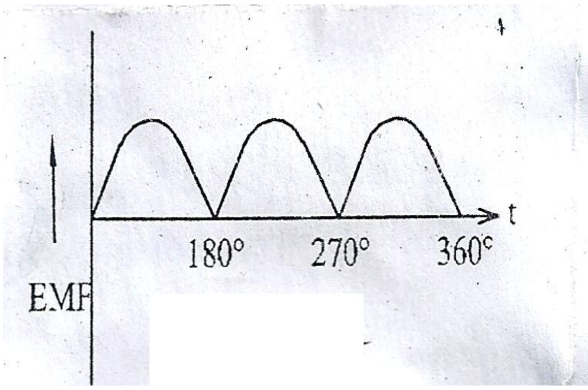


Fig.4.16

Shows the connection of coil ends with split rings ‘a’ and ‘b’. In the first half revolution current flows along ABLMCD in the brush No.1 is contact with segment ‘a’ acts the positive end of emf and ‘b’ acts as the negative end.

As the coil assumes successive position in the field the flux linked with it changes. Hence emf is induced in it. Which is proportional to the rate of change of flux linkages ($e=Nd\phi/dt$). When the plane of the coil is at right angles to lines of flux. i.e when it is in position 1. Then flux linked with the coil is maximum but rate of change of flux linkage is minimum.

In this position, the coil sides AB and CD do not cut, the flux, They move parallel to them. Hence there is no induced emf is the coil. The angle of rotation (or) time will be measured from this position.

As the coil continues rotating, the rate of change of flux linkages increases, till position 3 is reached where ϕ =90`, the coil plane is horizontal to the flux line, the flux linked with the coil is minimum, nut the rate of change of flux linkage is maximum. Therefore maximum emf is induced in the coil at position 3. In the next quarter revolution from position 3 to position 5 (90` to 180`), the flux linked with the coil gradually increases, but the rate of change of flux linkages decreases. Therefore the emf induced is zero at position 5.

Now the coil moves from position 5 to position 7 (180` to 270`), the emf induced.

In the next half revolution the direction of current in the coil has reversed, as shown in fig.4.13 (b).

But at the same time, the position of segments a and b have also been reversed. The segment 'a' is coming in contact with brush no:2 and becomes negative end of induced emf. Again the current in the load Resistance flows in the same direction. i.e. from L to M. The current is unidirectional current and is shown in fig. The AC current induced in the coil is converted into unidirectional current due to rectifying action of split rings. (also called as commutator)

The unidirectional current is shown in fig.4.14 To minimize the ripple in DC current the number of cols in the armature should be increased.

4.6 EMF EQUATION OF A GNERATOR

Let	ϕ	=	flux / pole in weber
	Z	=	Total number of armature conductors
		=	No. of slots x No. of conductors / slot
	P	=	No. of generator poles.
	A	=	No. of parallel Paths in Armature
	E	=	emf induced in any parallel path in armature.
	A	=	2 for wave winding
	A	=	P for lap windings

According to Faraday's law of Electromagnetic induction, the average emf in each conductor is equal to the rate of change of flux in webers per second.

$$E = N.\ d\phi/dt \text{ volt}$$

Now flux cut / conductor in one revolution $d\phi = \phi P$ Wb

No. of revolution / Second = N/60

Therefore time for one revolution, dt = 60 / N Second.

Therefore EMF induced in each conductor = $d\phi / dt$

$$= \phi P / 60 / N = \phi PN / 60 \text{ volt}$$

No of Armature conductors connected in series in each parallel path = Z/A

Therefore emf induced in DC generator.

$$E = \text{emf induced in Each conductor} \times \text{number of conductors in each parallel path}$$

Emf induced in DC generator $E_g = \phi$ P ZN / 60A

$$E_g = P\phi ZN / 60\ A$$

4.7 CHARACTERISTICS OF GENERATORS :

There are three most important characteristics or curves of a dc generator.

1. No. load saturation characteristic (E_o/I_f). It is also known as Magnetic characteristics on open-circuit characteristic (O.C.C). It is relation between no load generated emf in armature E_o and field (or) exciting current (I_f) at a given fixed speed.
2. Internal (or) Total Characteristic (E/I_a).
   It gives the relation between the emf E actually induces in the Armature and the Armature current I_a.
3. External Characteristics (V/I_L)
   It is also referred to as performance characteristic (or) sometimes voltage-regulating curve. It gives relation between that Terminal voltage V and the load current I_L.

1. LOAD CHARACTERISTICS OF DC SHUNT GENERATORS.

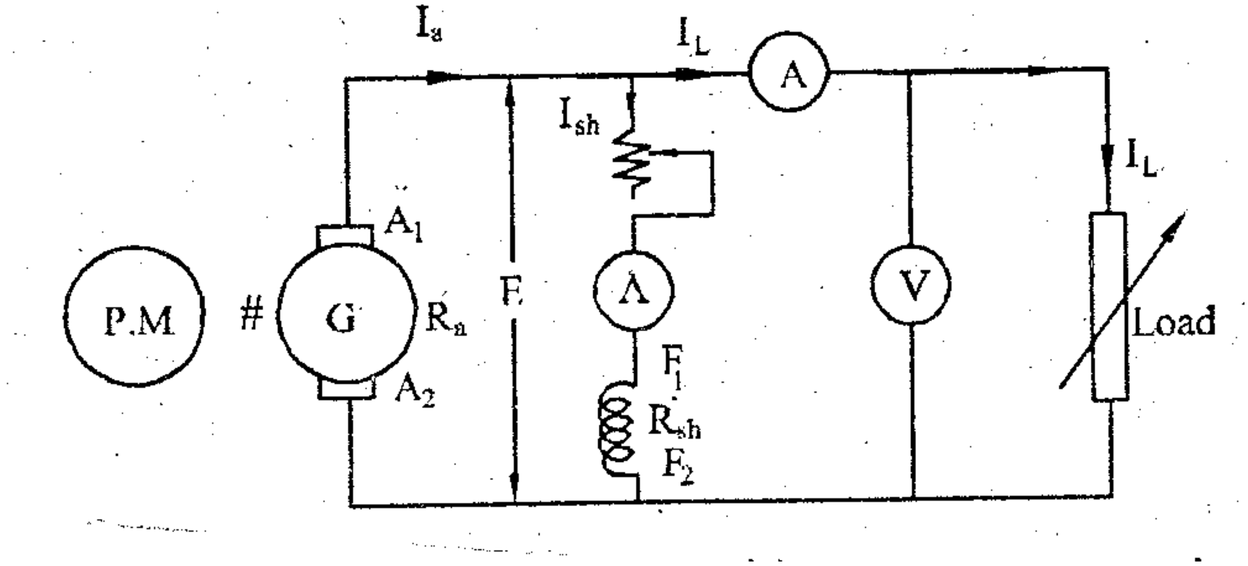


Fig.4.17 Load Characteristics DC Shunt Generator

Fig.4.17 Shown the circuit diagram of DC shunt generators to find out the load characteristics. In this circuit, the armature, field and load are connected in parallel to measure the terminal voltage a voltmeter is connected as shown in fig (4.17) to measure the field current and load current two ammeters are connected as shown in the circuit diagram.

In shunt generator $I_a = I_L + I_{sh}$

$$E = V + I_a R_a$$

The generator is started with the help of prime movers and is run at rated speed. Adjust the field Rheostat, s that the voltmeter reads the sated voltage. Then by keeping the field current constant, vary the load current, and note the Terminal voltage for each value of load current. For each value of terminal voltage, the induced EMF and armature current are calculated.

Then plot the load current on X-axis and Terminal voltage on Y-axis in the graph. We get the external characteristics curve as shown in fig4.17.

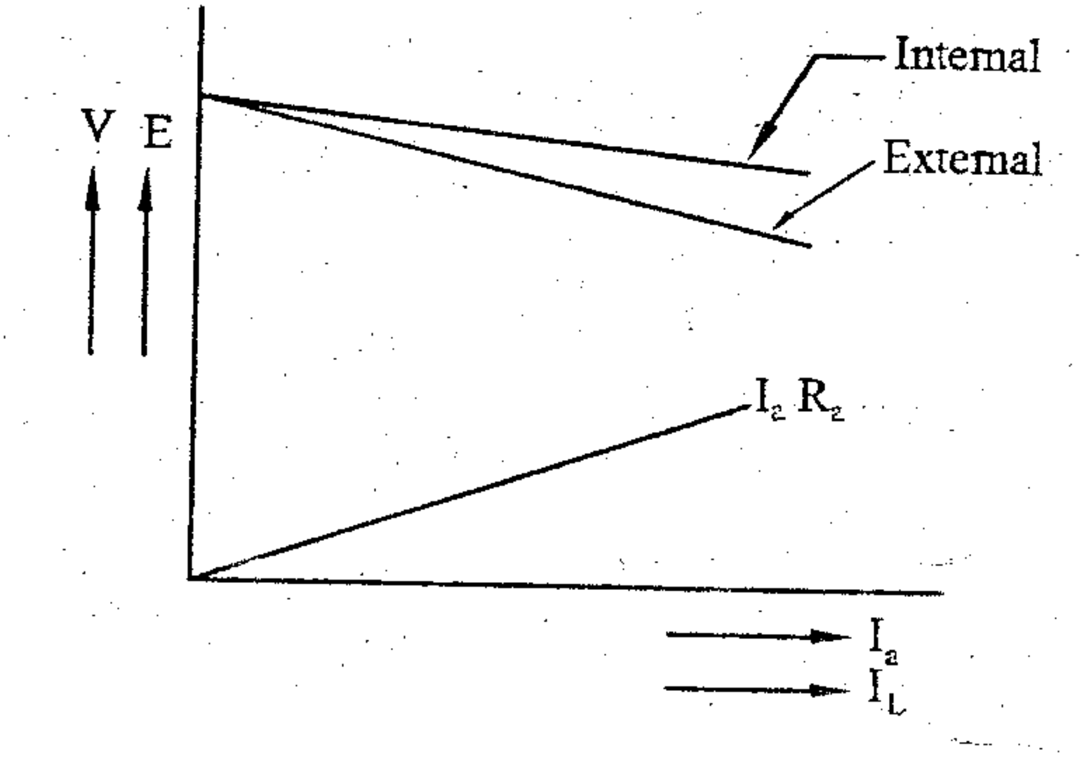


Fig:4.18, Load current Vs Terminal Voltage

Then plot the armature current in x-axis and induced emf in y-axis in the graph. We get the internal characteristics curve as shown in fig from the characteristic, we find that, when the current increases, the terminal voltage decreases. The voltage is reduced due to armature resistance drop and armature Reaction Effect.

When the current increases above the full load current, the voltage is reduced and becomes zero. Hence shunt generator has drop ring characteristics.

4.8.1. LOAD CHARACTERISTICS OF DC SERIES GENERATOR

The above fig4.17 shown the circuit diagram of DC series generator. Using this circuit diagram the load characteristics are drawn. Since the Armature, and series field are connected in series, the current through the armature, series field winding and the load current are same.

$[I_a = I_{se} = I_L]$

To measure the terminal voltage a voltmeter is connected. An ammeter is connected to measure the load current. The generator is started with the help of a prime mover and is allowed to run at its rated speed. Adjust the load step by step and also note terminal voltage and load current for every step, for each reading calculate the emf induced in the generator by using the formula.

$[E = V + Ia (Ra + Rse)]$

Plotting the load current, armature current, in x-axis and terminal voltage, and induced emf on y-axis. Now plot the internal characteristics [E Vs Ia] and external characteristies [V Vs I_L] are shown in fig.

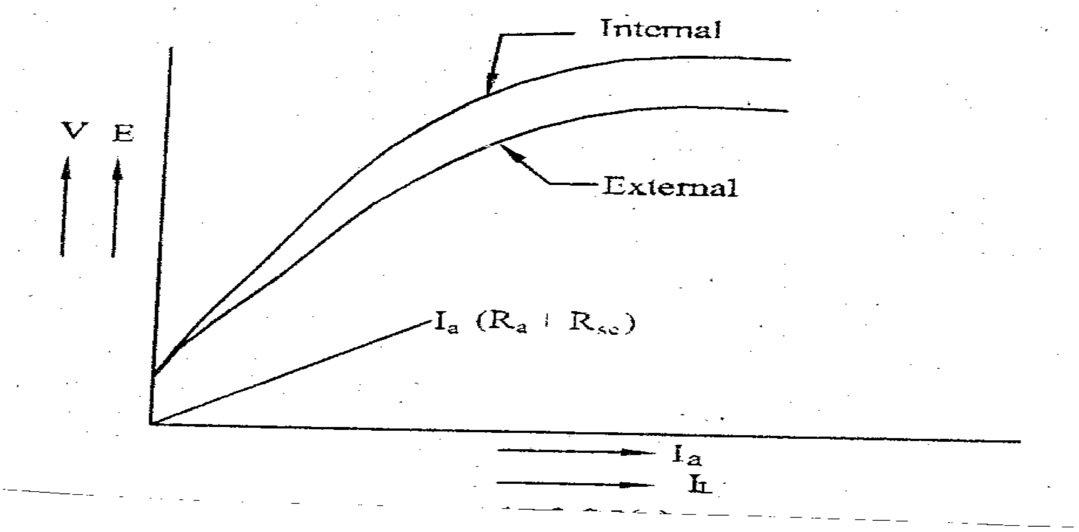


Fig.4.19

Since the field winding is in series with the load at no load (I_L=0) the induced emf in zero. (That is I_L=0, I_{se} also zero). Hence the emf is also zero, when the load current increases the field current also increases, which in turn increases the flux. So the terminal voltage and emf are also increased. Hence the series generator has rising characteristics. But after saturation of field, the voltage is not increased, even though the load current increases.

After the saturation, when the load current increases the voltage is reduced due to the armature reaction effect and armature resistance drop.

4.8.2 LOAD CHARACTERISTIC OF COMPOUND GENERATOR

In a compound generator, both series and shunt windings are combined as shown in fig (4.20). The shunt winding can be connected either across the armature only (short-shunt) (or) across the Armature plus series field [long shunt].

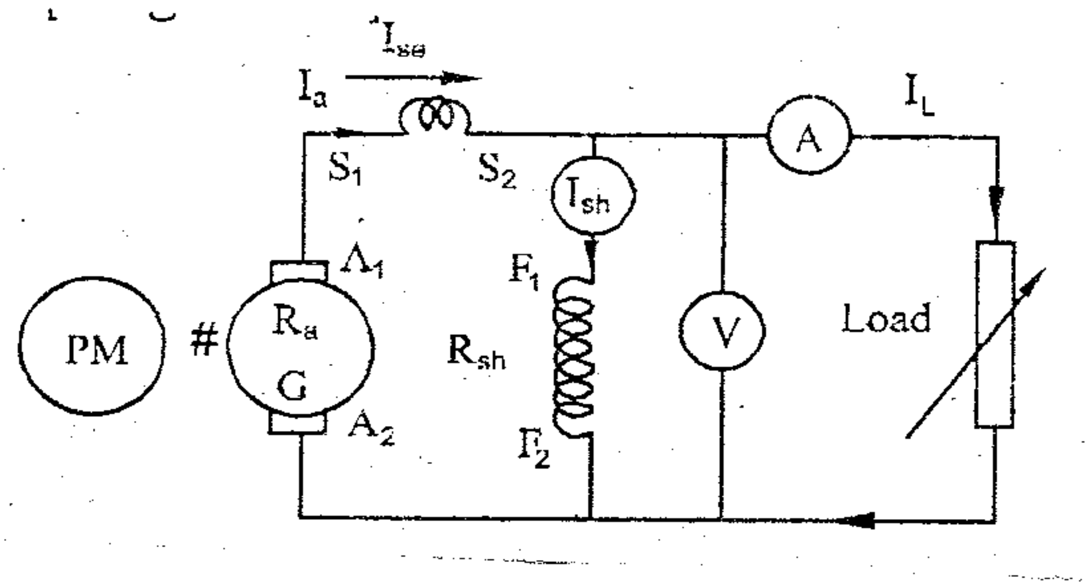


Fig 4.20 Long shunt

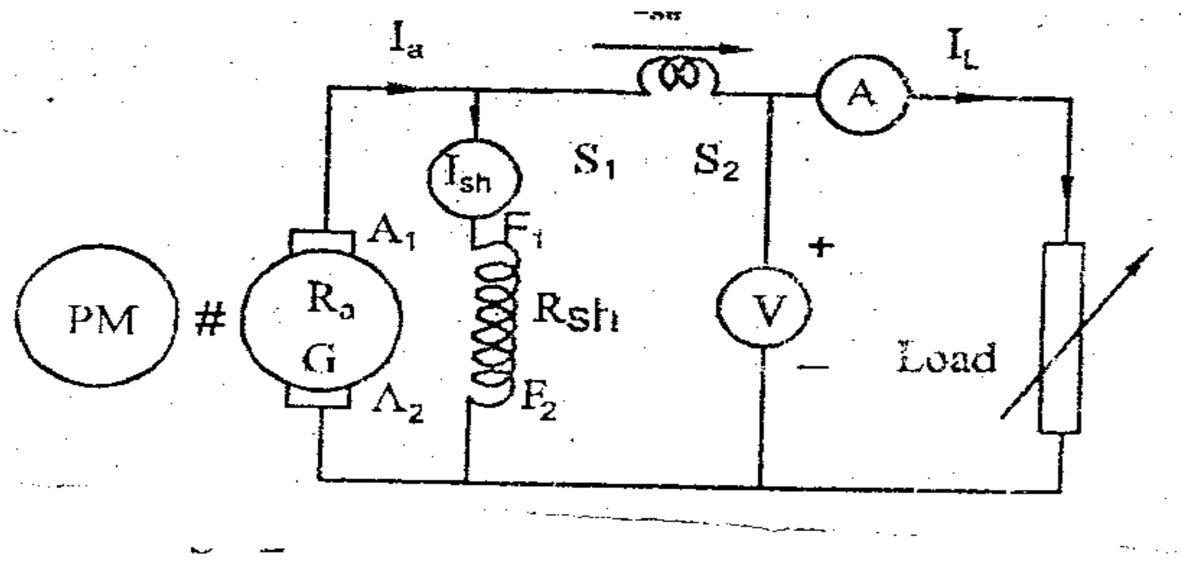


Fig 4.21 short shunt

The circuit diagram for drawing the internal and external characteristics of compound generator is given in Fig. 4.20 and Fig.4.21.

The generator is started with the help of a prime mover and is allowed to run at its rated speed. Adjust the load step by step and also note terminal load current and field current for every step for each reading calculate the emf induced in the generator by using the formula

$$E = V + Ia\ Ra + Ise\ Rse$$

The plotting load current, armature current in x-axis and terminal voltage and induced emf on y-axis, Now plot internal characteristics [E Vs Ia] and External Characteristics [V Vs I_L] as shown in Fig.4.22 & 4.23.

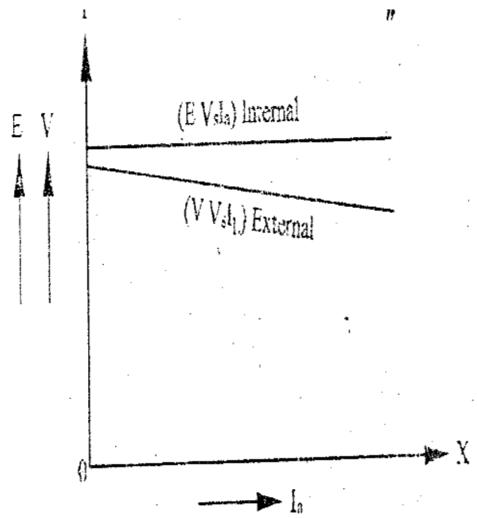


Fig 4.22

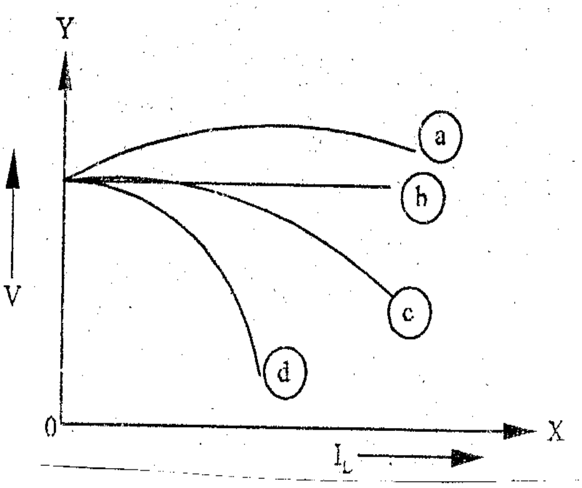


Fig 4.23

In fig various compound generator for the varies load characteristics of DC generator.

4.9 DC MOTOR

INDRODUCTION:

A DC Machine may be operated either as generator or as motor. The parts and construction of DC motor are some as that of DC generator. Hence DC motor converts electrical energy into Mechanical energy.

4.9.1 DC MOTOR PRINCIPLE OF OPERTION

An electric motor is a machine which converts electrical energy into mechanical energy. Its action is based on the principle that when a current carrying conductor is placed in a magnetic field, it experiences a mechanical force whose direction is given by Fleming's left hand rule and whose magnitude is given by F = BIl Newton.

Where

F	=	Force produced on the conductor in Newton
B	=	Magnetic flux density in web / m^2
L	=	Length of conductor in the magnetic field
I	=	The current flowing through the conductor in ampere.

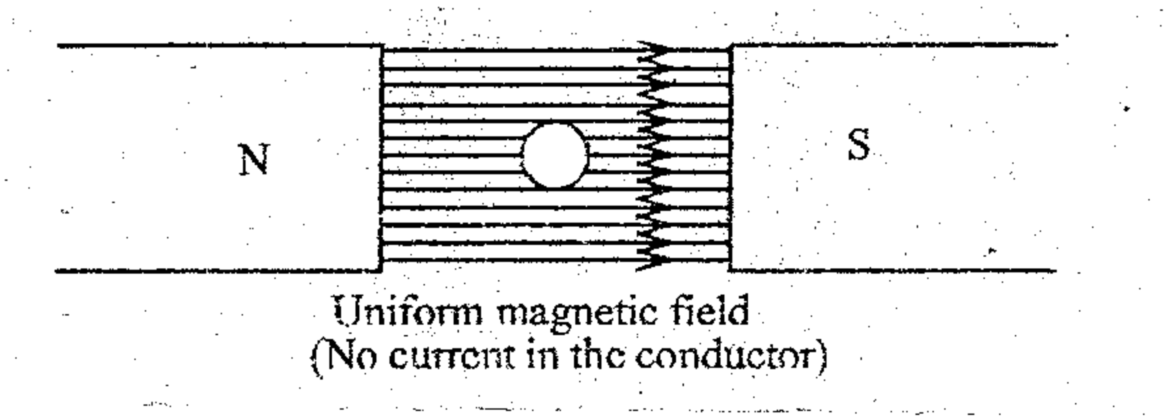


Fig 4.24

To understand the principle of operation of DC motor, let us consider a two pole motor.

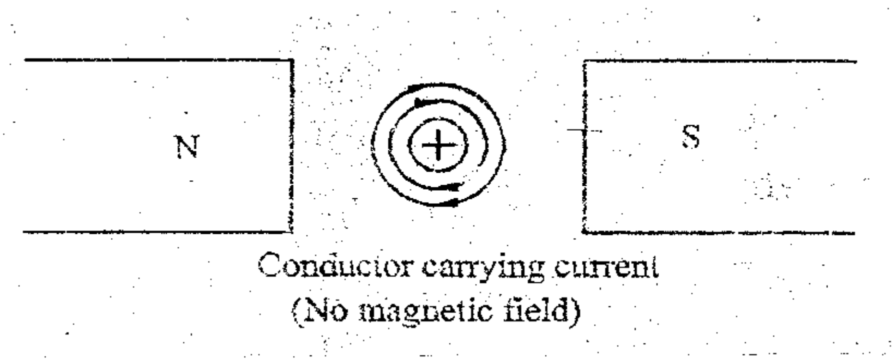


Fig 4.25

Fig 4.24 shows an uniform magnetic field in which a straight conductor carrying no current is placed. The direction of magnetic flux line is from north to south pole.

Now assume there is no exciting current flow through the field winding and DC current is sent through the conductor. Let the conductor carry the current away from the observer (+). It produces a magnetic flux lines around the in clockwise direction as shown in fig 4.25 There is no movement of the conductor during the above two conditions.

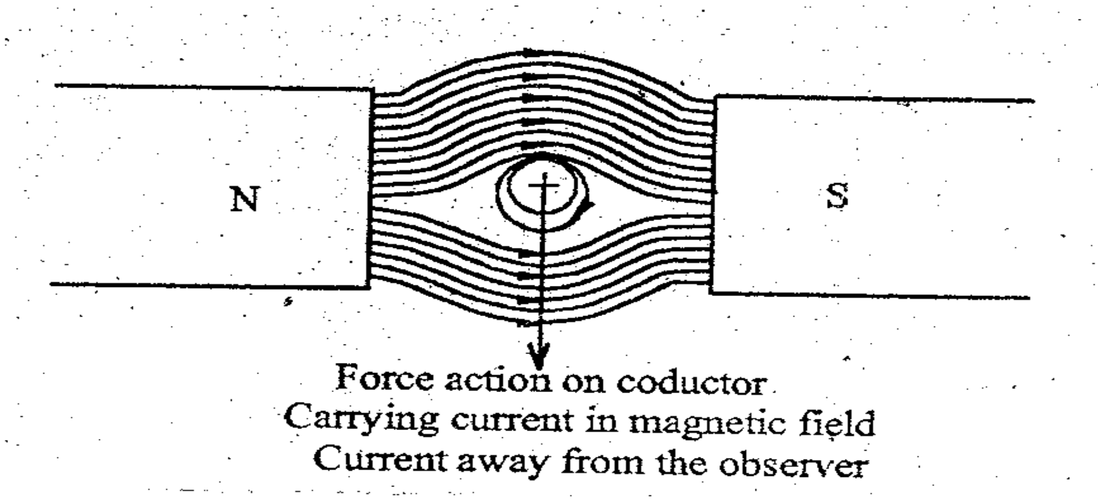


Fig 4.26

In fig 4.26 the current carrying conductor is placed in the magnetic field. The due to the current in the conductor aids the main field above the conductor, but opposes the main field below the conductor. Hence the flux strengthens above the conductor and weakens below the conductor. It is found that a force acts on the conductor. Trying to push the conductor downwards as shown by the arrow. The conductor is pushed from high flux density to low flux density.

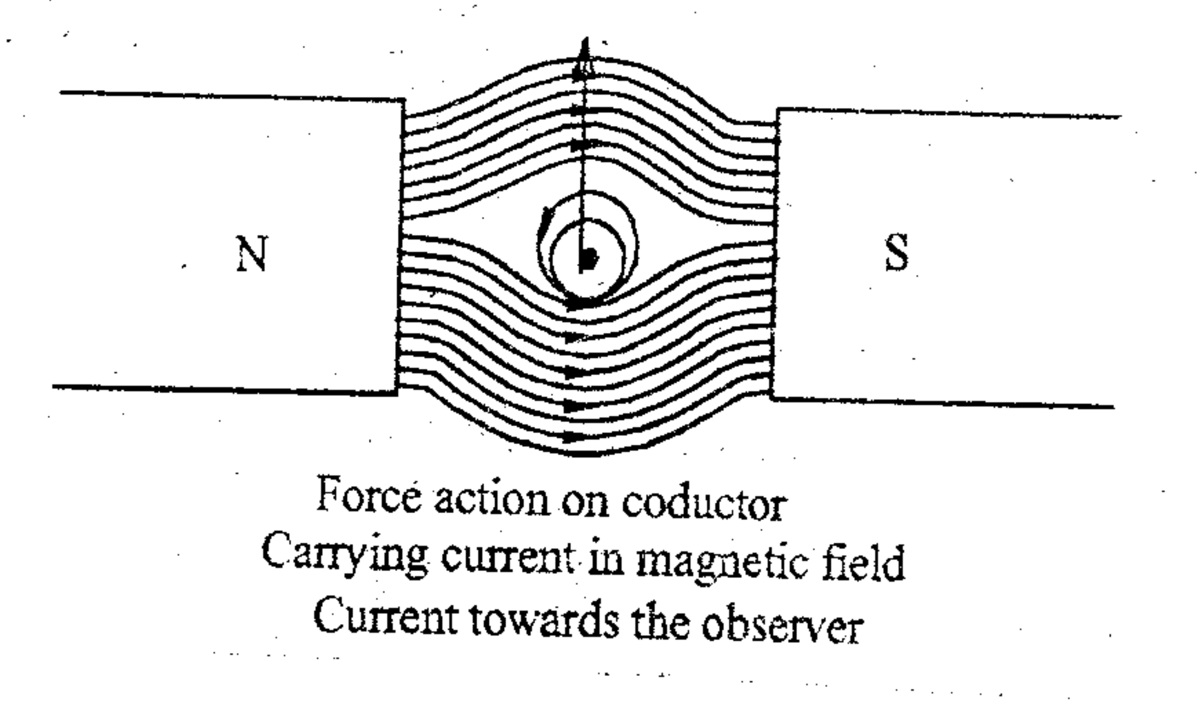


Fig:4.27

If the current in the conductor is reversed (current towards the observer (O), the strengthening of flux line occurs below the conductor and the conductor will be pushed upward as shown in fig 4.28

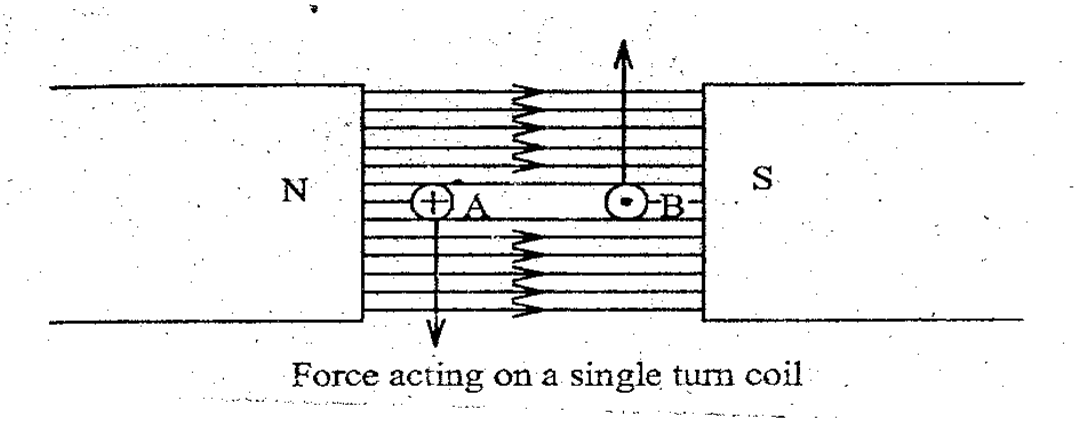


Fig.4.28

New consider a single turn coil carrying current as shown in fig 4.27 the coil side 'A' will be forced to move downwards, where as the coil side 'B' will be forced to move upwards. The forces acting on the coil sides 'A' and 'B' will be of same magnitude, but their direction is opposite to one another. As the coil is wound on the armature core, which is supported by the bearings, the armature will now rotate the direction of rotation is found out by fleming's left hand rule.

4.9.2 **TYPES OF MOTORS**.

The Dc motors are classified as

1. DC shunt motor
2. DC series motor
3. DC compound motor.

1. **DC SHOUNT MOTOR :**

In DC shunt motor, the field winding is connected in parallel with the armature as shown in fig.4.29 The field winding has a large number of turns and smaller crossection area. Since the field current is small, the field power loss is also small.

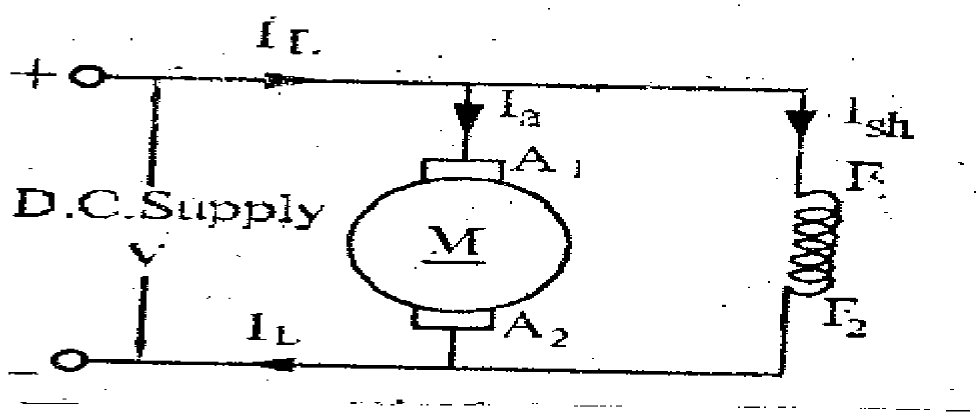


Fig.4.29

Voltage Equation of DC shunt motor is

$V = E_b + Ia\ Ra$ + Brush drop

Armature current $Ia = I_L - Ish$

Where

E_b = Back emf of motor

V =Applied voltage

Ia =Armature current

Ra =Armature Resistance.

2. **DC SERIES MOTOR** :

In DC series motor, the field winding is connected in series with the armature as shown in fig The series field winding gets the input current [$I_L = Ia = Ise$]. The series field winding has large cross – sectional area and few number of turns, also the field winding has low resistance.

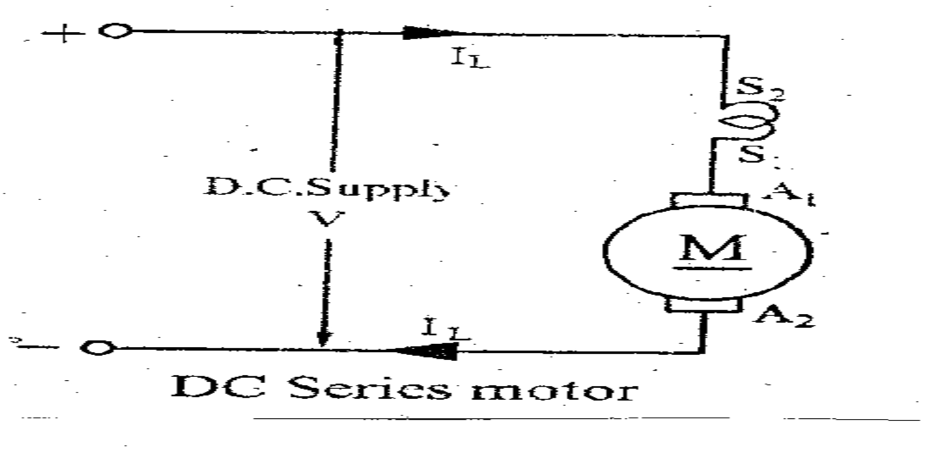


Fig. 4.30

The voltage equation on the DC series motor

$V = E_b + Ia\ Ra + Ise\ Rse$ + Brush drop

Since $Ia = Ise = I_L$

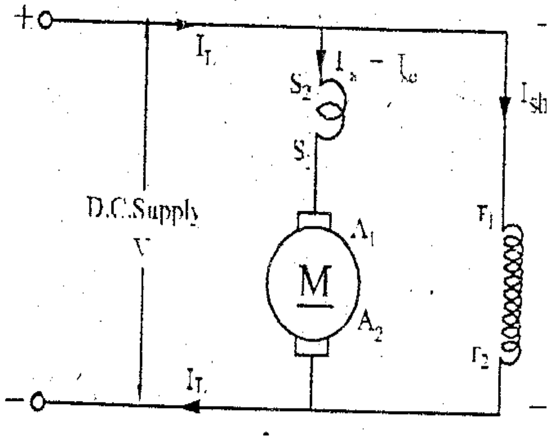
Therefore $V = E_b + Ia\ [Ra + Rse]$ + brush drop

Where

Rse	=	Series field current
Ia	=	Armature current
Ra	=	Armature Resistance
Rse	=	Series field resistance
V	=	Applied voltage
E_b	=	Back emf induced

3 .**DC COMPOUND MOTOR :**

In compound motor, both series field and shunt field windings are connected with the Armature



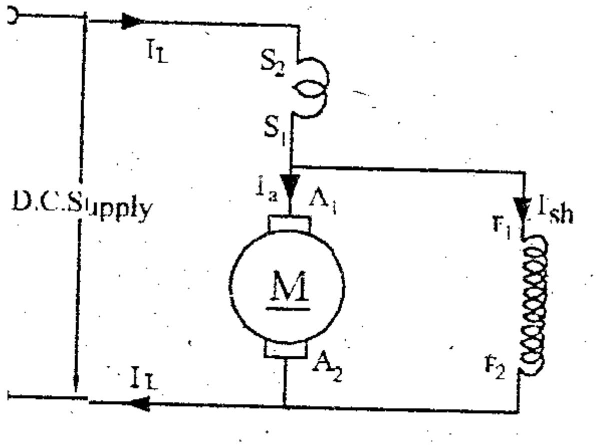


Fig. 4.31 Long shunt DC compound motor   Fig 4.32 Short shunt DC compound motor

In fig4.31 the shunt field winding is connected across the series combination of armature and series field winding and this arrangement is connected across the supply. It is called as long shunt DC compound motor. In fig.4.32 the series field winding is connected in series with the parallel combination of shunt field winding and armature winding, and this arrangement is connected across supply. It is called short shunt DC compound motor

The voltage Equation

$V = E_b + Ia\ Ra + Ise + \text{Brush drop}$

Ia Ra = Armature resistance drop

Ise Rse = series field drop

Long shunt compound motor

Ia = Ise

Short shunt compound motor

$Ise = I_L$

Compound motor .

$Ia = I_L - Ish$

In DC compound motor if the shunt field winding and series field winding are connected such that their fluxes add with each other they are called cumulative compound DC motor.

If they are connected such that flux produced by the series field winding opposes the flux produced by shunt field winding then the motor is called differential compound motor.

4.10 BACK EMF (OR) COUNTER EMF

When DC supply is given to DC motor its armature starts rotating. The armature rotates and cuts the static magnetic flux produced by the field magnets. Therefore emf is induced in the armature conductors (Faraday's law's of Electromagnetic induction). As per Lenz's law this induced emf opposes the supply voltage. Hence the emf induced in the armature is called Back emf (or) counter emf (E_b).

Back emf induced in the armature of a D.C. motor.

$E_b = P\phi ZN / 60A$

Where

P	=	Number of poles
ϕ	=	Flux per pole in webers
Z	=	No of conductors in the armature
N	=	Speed in RPM
A	=	No of parallel paths

Ia Ra is the voltage drop in the armature circuit

$E_b = V - IaRa$

Voltage drop in the armature circuit

$Ia\ Ra = V - E_b$

Therefore Armature current $Ia = V - E_b/Ra$

Significance of back emf :

The Back emf in DC motor regulates the flow of armature current, i.e., Automatically changes the armature current to meet the load requirements and it makes a motor self regulating

4.11 TORQUE – SPEED RELATIONSHIP

DC SHUNT MOTOR.

$N \alpha E_b/\phi$ i.e., $N \alpha V - Ia\ Ra/\phi$

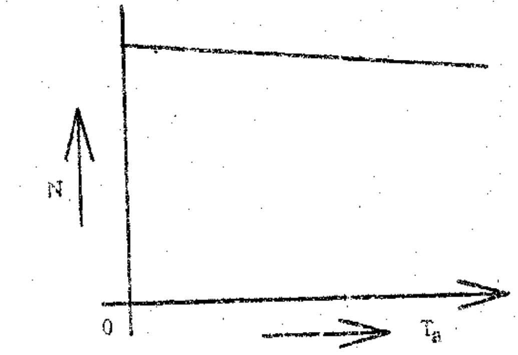


Fig.4.33 Speed Vs torque

In shunt motor, ϕ is constant, $N \alpha V – Ia Ra$ ------(1)

$T \alpha \phi$ Ia

Since ϕ is constant, T & Ia

Therefore $Ia = KT$ ---- (2)

Where K is a constant

Substitute equation (2) in equation(1)

$N \alpha V-(KT) Ra$

From the above equation when the torque increases the speed decreases.

DC SERIES MOTOR :

In any DC motor $N \alpha V-Ia Ra/\phi$

If Ia Ra drop is negligible

$N \alpha V/\phi$ ----------- (3)

We know $T \alpha \phi Ia$

In series motor $Ia \alpha \phi$

Therefore $T \alpha \phi.\phi$

$T \alpha \phi^2$

Therefore $\phi^2 = T$

$\phi = \sqrt{} T$ ----------------(4)

Substitute the equation (4) in (3)

$N \alpha V\sqrt{}T$

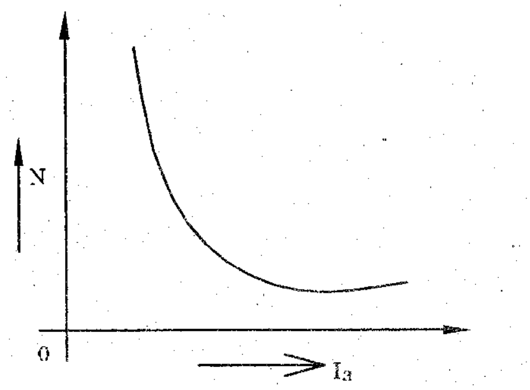


Fig. 4.34Speed – torque curve

From the above equation speed is inversely proportional to torque. Hence in DC series motor, the torque increases with decreases of speed.

4.12 SPEED CONTROL OF DC MOTOR

Speed control of DC shunt motor

1. Armature control method.

In this method based on by varying the voltage available across the armature. Hence the back emf and speed of the motor can be changed. This is done by inserting a variable resistance in series with the armature as shown in fig.

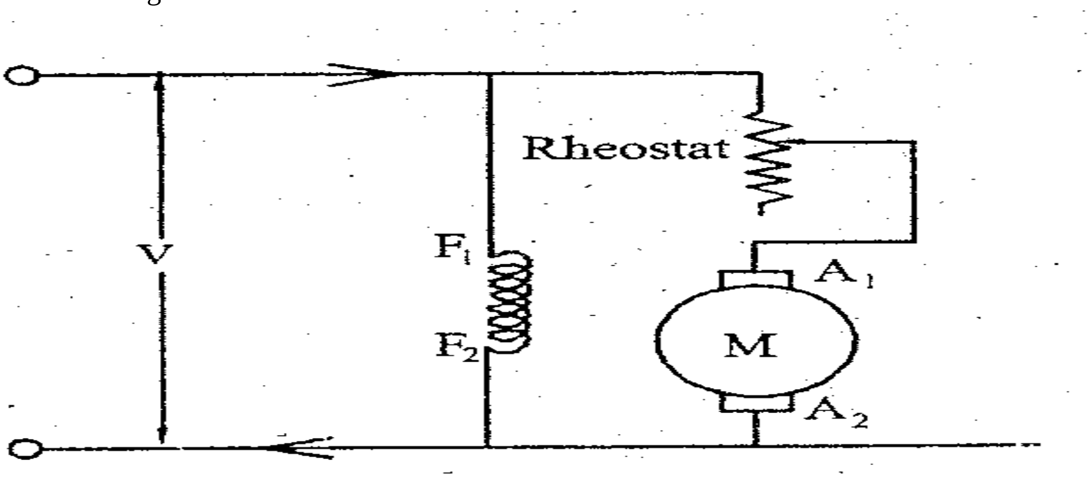


Fig 4.35 Armature control method

2. FIELD CONTROL METHOD

The speed of a DC motor is inversely proportional to the flux per pole (N&1/Q). Hence by varying the flux, the speed can be varied. The flux per pole of a DC motor can be charged by varying the field current. This field current can be changed by shunt field Rheostat as shown in fig.

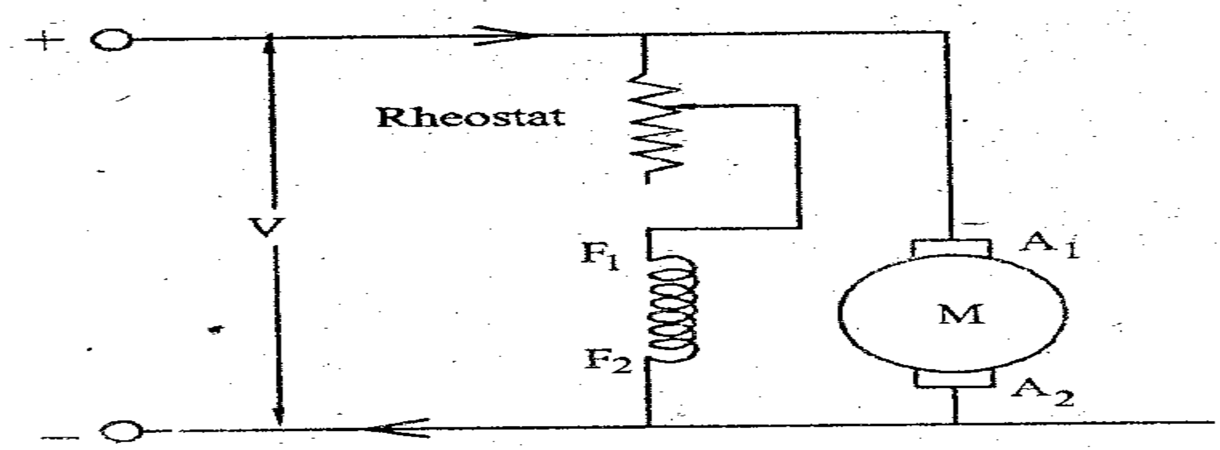


Fig 4.36 Field control method in shunt motor

1. ARMATURE RESISTANCE CONTROL METHOD

In this method a variable resistance is directly connected in series with the supply as shown in fig. this reduces the voltage available across the armature and hence the speed falls. Pay changing the value of variable resistance. Any speed below the normal speed can be obtained. This method is mostly used to control the speed of DC series motor.

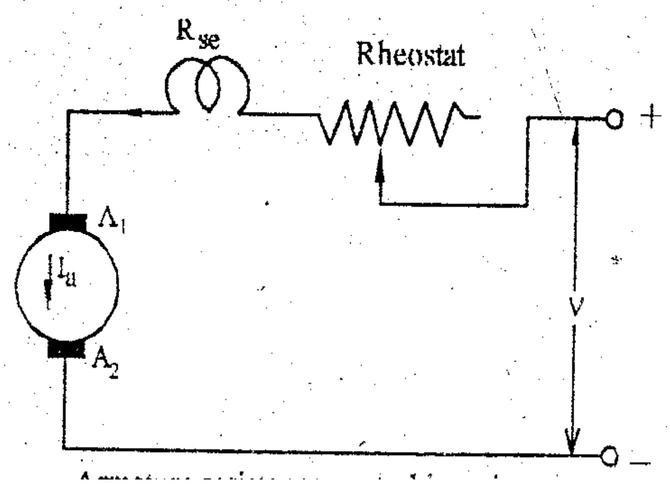


Fig 4.37

4.12.2 **FIELD CONTROL METHOD**

In field control method further 3 types.

1. Field Diverter Method
2. Armature Diverter Method
3. Tapped Field Control Method

4.12.1.1**FIELD DIVERTER METHOD.**

A variable resistance called field diverter is connected in parallel with the series field winding as shown in fig 4.38. Any desired amount of current can be passed through the diverter by adjusting its resistance. Hence the flux can be decreased and hence the speed of the motor is increased (N & 1/Q). Hence this method can provide speed above normal speed.

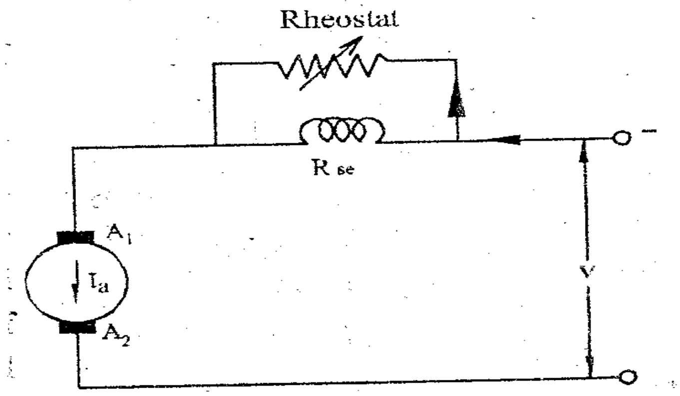


Fig. 4.38 Field diverter method.

4.12.1.2**ARMATURE DIVERTER METHOD**

In order to obtain a speed below the normal speed a variable resistance called armature diverter is connected in parallel with the armature as shown in fig. 4.39

Hence by adjusting the armature diverter, any sped laver than the normal speed can be obtained.

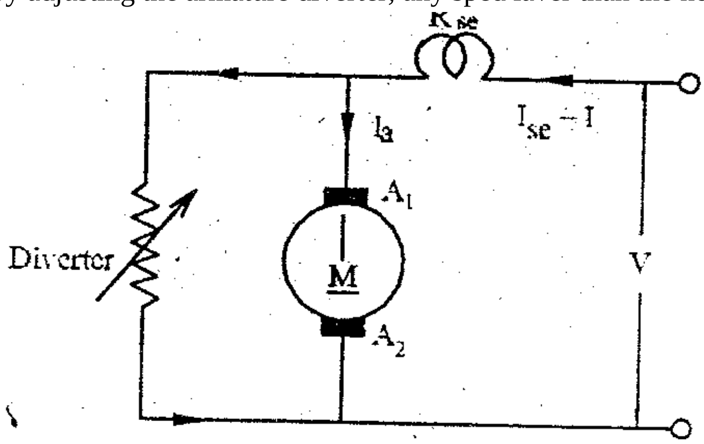


Fig 4.39 Armature diverter method

4.12.1.3**TAPPED FIELD CONTROL METHOD.**

In this method, the flux in reduced and hence the speed is increased (N & 1/Q) by decreasing the number of turns of the series field windings as shown in fig. 4.40The switch can short circuit any part of the field winding, they decreasing the flux and razing the speed. When the field windings are in full turns, the motor runs at normal speed and when the field turns are cutout speeds higher than normal sped are obtained.

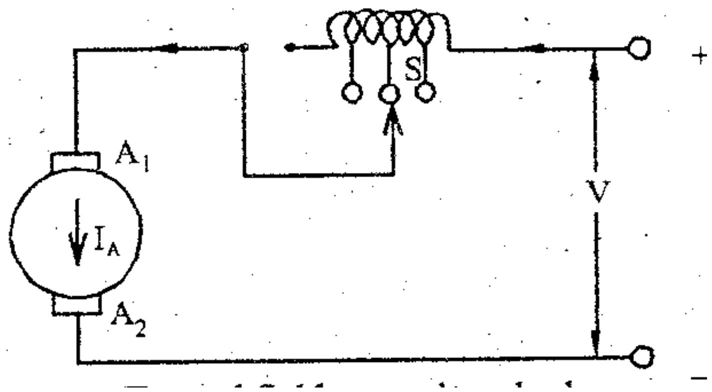


Fig 4.40. Tapped field control method

STARTER FOR DC MOTOR :

In order to start DC shunt motors and DC compound motors, the following starters are used.

1. 3-Point Starters
2. 4-Point Starters

4.13 3-POINT STARTERS

This starter is used to start DC shunt motor and compound motor. The circuit diagram of three point starter as shown in fig.4.41. Since the motor is connected to supply terminals through three terminals L, A, and F it is called three point starter.

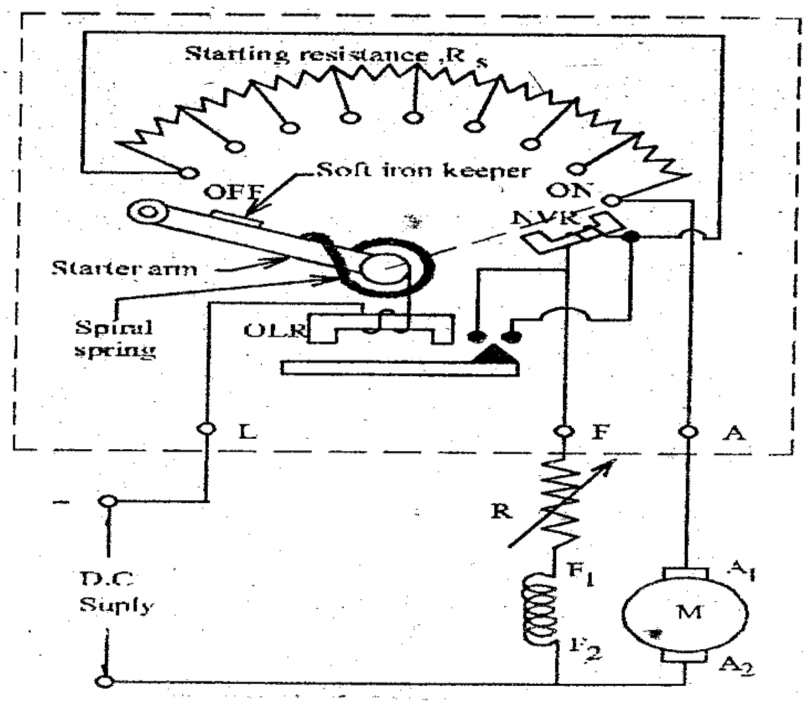


Fig.4.41

In this starter, the resistor elements are mounted behind an insulation board. The tapping points of starting resistance are brought out to a number of studs. The handle of the starter is fixed to a point so as to be moved over the studs against a spring tension.

When the starter handle is moved to the first stud a reduced voltage is applied to the armature, due to drop in the resistor elements. Hence the starting current is limited to safe value. At the same time full voltage is applied across the field, and this produces normal flux. When the starter handle is moved towards right, the resister elements are cut out one by one and the voltage applied to the armature increases step by step when all resistances are cut out, the handle is in on position, now full voltage is applied across the armature terminals. A soft iron piece is attached to the handle now the handle is attracted by the no volt release coil.

NO VOLTAGE RELEASE :

If consists of an Electromagnet (NVR). If is connected in series with the shunt field circuit and hence it carries shunt field current, so this is energized is holds the handle is "on" position. In the case of failure get de-energized and handle flies back to "off" position.

OVER LOAD RELEASE :

This is also consists of an electromagnet. The electromagnet is energized by the line current. When the load on the motor is increased, the magnetizing force produced by this coil is sufficient to left the movable iron. The morale iron in turn short circuits the terminals of the no volt release. Hence the no volt coil is de energized and the starter handle returns to 'off' position. This overload release protects the motor against over loads.

DEMERITS :

A motor can be run for a higher speed than normal speed by reducing the field current. The reduced field current may not produce enough magnetic force to hold the handle "ON" position. So the handle returns to the "OFF" position. This is the disadvantage of three point starter. It is eliminated in four point starter.

4.1.4 **FOUR POINT STARTER :**

Four point starter is used for starting shunt and compound motors.

A motor can be run for a higher speed than the normal speed by reducing the field current, the reduced field current may not produce sufficient magnetic force to hold the handle in "ON" position. So the handle returns to the "OFF" position. This is the disadvantage of three point starter. This draw back in overcome in four point starter. To avoid this draw sack in four point starter the no volt coil arranged such that NVR supply is independent of the shunt field current. The no volt coil depends upon the supply voltage there the no volt release coil in series with protective resistor "RP" is connected across the terminals L + L- as shown in fig.

There are four terminals namely

1. L+ (Line Plus)
2. L – (Line Minus)
3. A (Armature)
4. F Field

In this starter, the resistor element are mounted behind an insulating board. The tapping point of starting resistances are brought out to a number of studs. The handle of the starter is fixed to a point so as to be moved over the studs against a spring tension.

When the starter handle is moved to the first stud, a reduced voltage is applied to the armature due to drop in the resister elements. Hence the starting current is limited to safe value. At the same time full voltage is applied across the field and this producer normal flux.

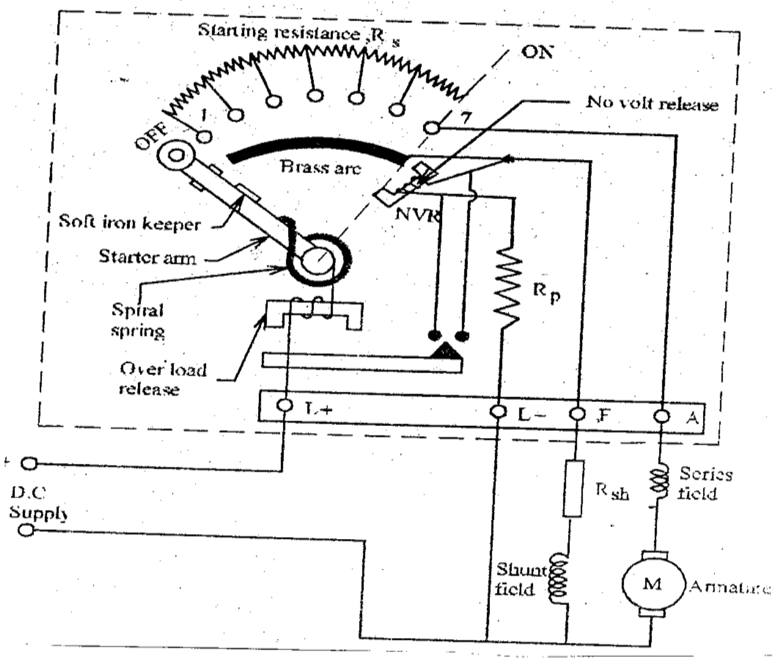


Fig.4.42

When the starter handle is moved towards right the resistor elements are cut out one by one and the voltage applied to the armature increases step b step when all resistances are cut out the handle is in “ON” position. Now full voltage is applied across the armature terminals. A soft iron piece is attached to the handle. Now the handle is attracted by the no volt release coil.

NO VOLT RELEASE :

If consists of an electromagnet (NVR). If is connected directly across the supply line through a protective resistor Rp. It holds the handle in “ON” position in the case of failure of supply or the voltage is very low the no volt coil gets de energized and handle files back to “OFF” position.

OVERLOAD RELEASE :

This also consists of a electromagnet. The electromagnet is energized by the line current. When the load on the motor is increased, the magnetizing force produced by this coil is sufficient to lift the movable iron. The movable iron in turn short circuits the terminals of the starter handle returns of “OFF” position. Thus the overload release protects the motor against overloads.

4.1.5 **APPLICATION OF DC MOTORS**

1. DC SHUNT MOTOR :

This type of motors are used to drive centrifugal pumps, light machine tools, reciprocating pumps, wood working machines, paper mills drilling machines. Shunt motors are used where constant speed is required at low a starting torque.

2. DC SERIES MOTOR :

This type of motors are typed where high starting torque is required, such as electric trains, cranes, lefts and conveyors.

3. COMPOUND MOTOR :

This types of motors are used where intermittent high starting torque is required as rolling mills presser printing machines, punches shears, conveyors etc.,

4.16 **AC - MACHINES** :

4.16.1 **3ϕ ALTERNATOR CONSTRUCTION AND WORKING**

The term alternating current generator commonly refers to alternators. They operate on the principle of faraday's laws of electromagnetic induction. An alternator is an electrical machine which converts mechanical energy into alternating electric energy. They are also known as synchronous generators.

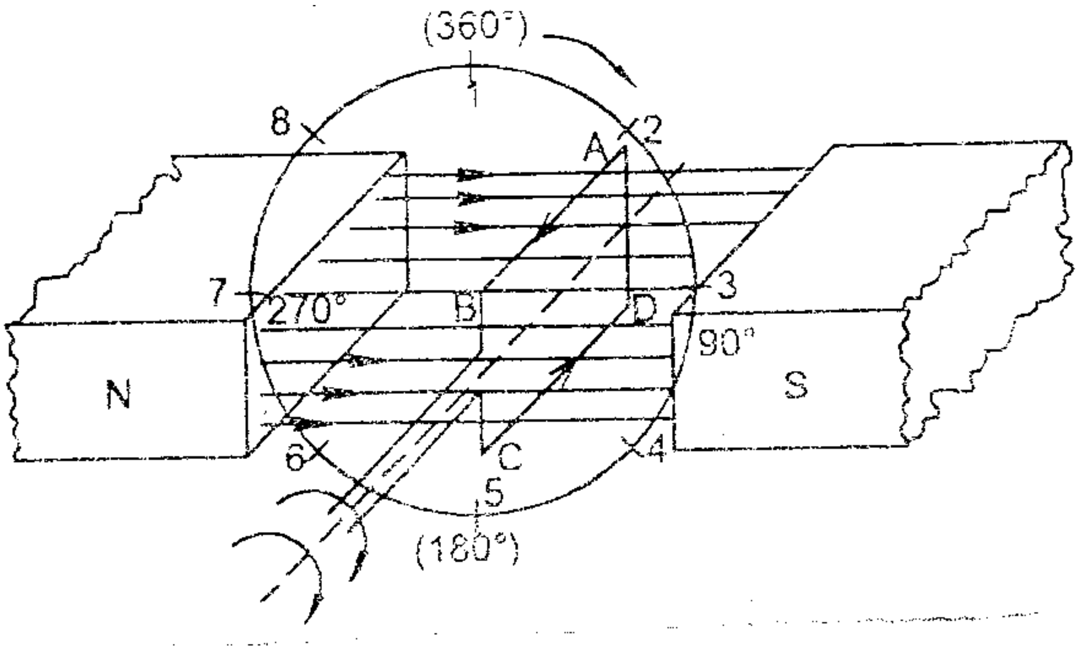


Fig.4.43

The working principle of an alternator or AC generator is similar to the basic working principle of a DC generator

The above fig helps you understanding how on alternator (or) AC generator works. According to the faraday's law of electromagnetic induction, whenever a conductor moves in a magnetic field EMF gets induced across the conductor. If the close path is provided to the conductor, induced emf causes current to flow in the circuit.

Let the conductor coil ABCD is placed in a magnetic field. The direction of magnetic flux will be form N pole to S pole. The coil is connected to slip rings, and the load is connected through brushes resting on the slip rings now consider the case 1 from above figure. The coil is rotating clockwise, in this case the direction of induced current can be given by Fleming's night hand rule and it will be along A-B-C-D.

As the coil is rotating clockwise after half of the time period the position of the coil will be as in second case of above figure. In this case, the direction of the induced current according to Fleming's right hand rule will be along D.C.B.A. It shows that the direction of the current changes after half of the time period that means we get on alternating current.

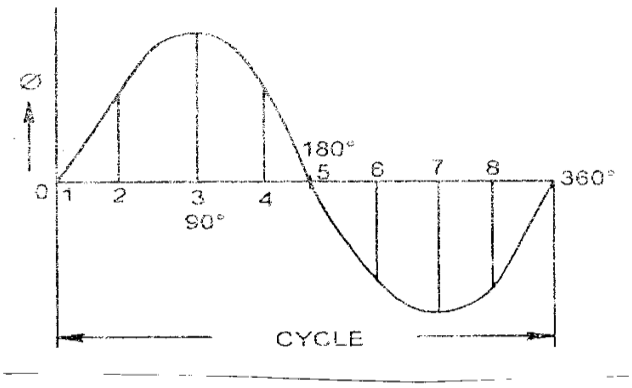


Fig.4.44

4.16.2 **CONSTRUCTIONAL DETAILS OF ALTERNATOR :**

According to the construction of rotor, alternators are classified into two types.

1. Salient Pole Alternator
2. Cylindrical type Alternator (Non Salient Pole)

1. **SALIENT POLE ALTERNATORS** :

Alternators driven at slow and moderate speeds are normally constructed with salient pole type of rotor. The prime movers used are water turbine (or) diesel engine.

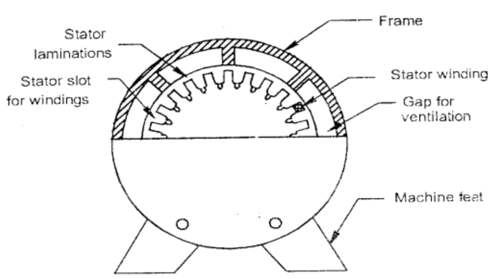


Fig.4.45

Main parts of the alternator. Obviously consists of stator and rotor , But the unlike other machines, in most of the alternators, field exciters are rotating and the armature coil is stationary.

Stator : Unlike in DC Machines stator of an alternator is not meant to serve path for magnetic flux. Instead the stator is used for holding armature winding. The stator core is made up of lamination of steed / alloys or magnetic room, to maize the eddy current losses.

Why Armature Winding is stationary in an alternator?

- At high Voltages, It easier to insulate stationary armature winding, this may be as high 30 Kv or more.
- The high voltage output can be directly taken out from the stationary armature, whereas, for a rotary armature, there will be large brush contact drop at higher voltages, also the sparking at the brush surface will occur.
- Field exciter winding is placed in rotor, and the low dc voltage can be transferred safely.
- The armature winding can be braced well. So as to prevent deformation caused by the high centrifugal force.

ROTOR : There are two type of rotor used in an AC generator / Alternators.

(i) Salient type rotor
(ii) Cylindrical type rotor.

4.16.3 **Salient Pole roto**r :

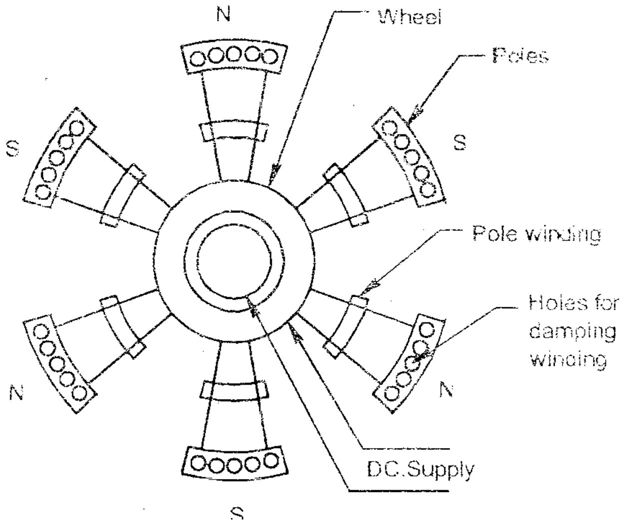


Fig.4.46

Salient pole type rotor is used in low and medium speed alternators. Construction of AC generator of salient type rotor shown in figure above. This type of rotor consists of large number of projected poles

(Called Salient pads) bolted on a magnetic wheel. These poles are also laminated to minimize the eddy current losses. Alternator and short in axial length.

Damper Windings :

Damper Windings are provided in the pole face of the Salient Pole rotor. Slurs are provided in the pole shoes. Copper bars are inserted in the slots and the ends of all the bars in both sides are short circulated by copper rings to have a closed circuit. This arrangement is called damper windings.

4.16.4 **Cylindrical type rotor (non salient pole rotor)**

Cylindrical type rotors are used in high speed alternators especially in turbo alternators. This type of rotor consists of a smooth and solid steel cylinder having slots along is outer periphery field windings are placed in these slots. The DC supply is given to the rotor winding. Through the slip rings and brushed arrangement. In this type of rotor the Damper winding is not necessary. The wind age loss is less.

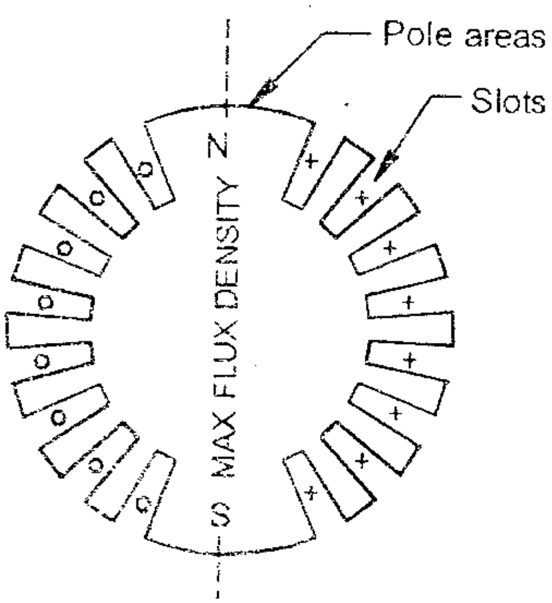


Fig.4.47

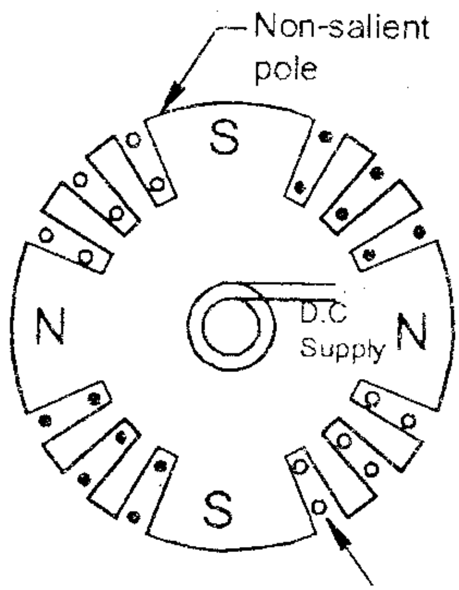


Fig.4.48

4.17 Relation between speed and frequency:

In an alternator, there exists a definite relationship between the rotational speed (N) of the Rotor, the frequent (f) of the generated emf and the number of poles P.

The emf in an armature conductor goes through one cycle in angular distance equal to take the pole pitch.

Let	P	-	total numb of magnetic poles
	N	-	Rotation speed of the rotor in rpm
	F	-	Frequency of generated emf in HZ

Since one cycle of emf is produced when a pair of poles passes past a conductor, the number of cycles of emf produce in one revolution of the rotor is equal to the number of pair of poles.

- No.of cycles / revolution = P/2
- No.of Revolution / Second = N/60
- Frequency = P/2 X N/60 = $\frac{PN}{120}$ HZ

$$F = \frac{PN}{120} \text{ HZ}$$

N is known is synchronous speed

4.18. Three phase Induction motor: (3ϕ) – Phase construction of 3ϕ Induction Motor:

An Induction Motor consists of two main parts namely stator and motor. There are two types of induction motor. They are squirrel cage and slip ring induction motor. This motor consists of two major parts.

Stator:

Stator of three phase induction motor is made up of number of slots to construct a three phase winding circuit which is connected to 3 phase AC source. The three phase winding are arranged in such a manner in the slots that they produce a rotating magnetic field after three phase AC supply is given to them.

Rotor :

Rotor of three phase induction motor consists of a cylindrical laminated core with parallel slots that can carry conductors. Conductors are a heavy copper or aluminums bar which fits in each slots and they are short circuited by the end rings. The slots are not exactly made parallel to the axis of the shaft but are slotted a little skewed because this arrangement reduces magnetic humming noise and can avoid stalling of motor.

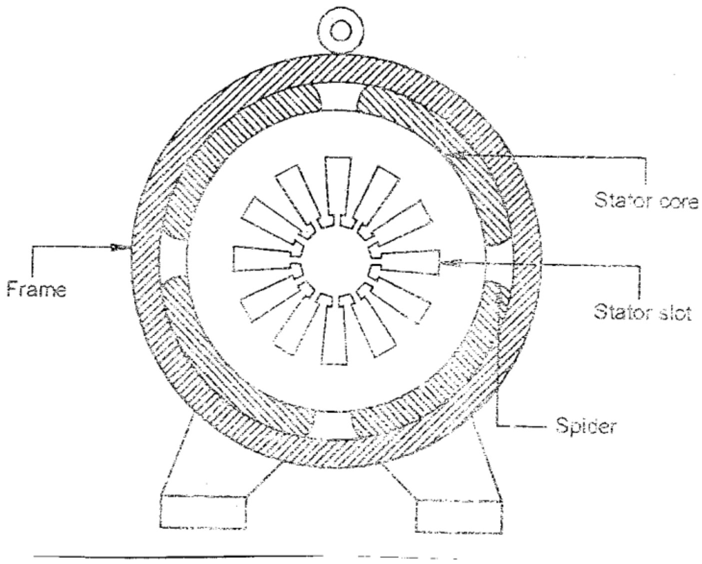


Fig.4.49

Rotor Construction :

Three phase induction motor are classified into two types according to their rotor construction

1. Squirrel Cage Rotor
2. Slip Ring Rotor

4.18.1 **Squirrel Cage Rotor**

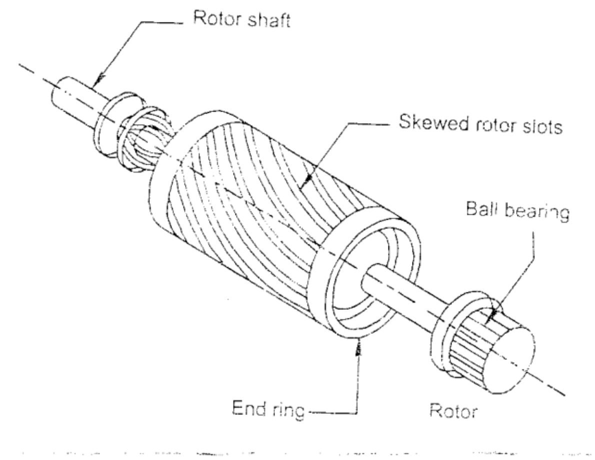


Fig.4.50

The rotors of motor also consist of laminated silicon steel punching, bolted together and mounted on a shaft. This core is cylindrical in shape with slots on the outside surface. The rotor winding consists of copper bars. One bar is placed in each slot. These slots are semi closed (or) totally closed. The rotor slots are slightly skewed and avoid is the magnetic locking. Hence in this type of rotors, it is not possible to add any external resistance in series with the rotor circuit for starting (or) speed control. This type of rotor is called squirrel cage induction motor.

4.18.2 **Slip Ring Rotor (Phase wound Rotor)**

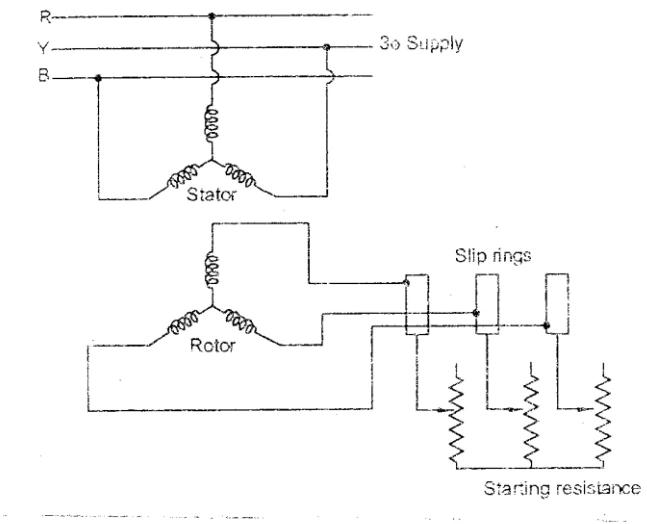


Fig.4.51

It is made up of steel laminations this type of rotor is provided with three phase double layer distributed winding this three phase windings is accommodated in the rotor slots. Each end of the three phase winding is connected in star, the remaining three terminals are brought out and they are connected to three slip rings theses slip rings are mounted on the shaft with insulation provided between each other. The brushes are so arranged to rest on the three slip rings. External star connected rheostat is connected to these slip rings. For slip rings high quality phosphor bronze is used.

At starting the external resistances are connected in series with the rotor and hence the starting torque of the motor is high.

The air gap between stator and rotor is always very small. For small machines the air gap is from 0.35mm to 0.65 mm, for larger machine rating air cap is from 1.00mm to 1.5mm.

4.19 **Principle of Operation of Three Phase Induction Motor.**

The stator of a three phase induction motor consists of a balanced three phase winding when the stator windings are fed by a three phase supply, then a magnetic field notating at synchronous speed is produced. This flux cuts the rotor conductors and an emf is induced in the rotor conductors according to faraday’s lays of electromagnetic induction. The frequency of the induced emf is the same as the supply frequency as the rotor is stationary now. Its magnitude is proportional to the relative velocity between the flux and the conductors.

Since the rotor conductors are short circuited a current is flowing through the rotor conductors. Hence a magnetic field is produced in the rotor conductors. One to the interaction of stator and rotor magnetic fields, the rotor begins to rotate in the same direction as that of the rotating magnetic flux and tries to catch up with rotating flux. But the sped or the rotor is lesser than that of the stator rotating magnetic field.

4.20 **Methods of Starting of Three Phase Induction Motor** :

In order to reduce the starting current of induction motor various methods for used in 3 phase induction motors are.

1. D.O.L. Starter
2. Rotor Resistance
3. Auto Transformer Starter
4. Start delta Starter.

4.20.1 **D.O.L. STARTER**

D.O.L. Starter is otherwise referred to direct-on-line starter. Using this starter, motors are started with full line voltage across them push button type D.O.L Starter as shown in fig.

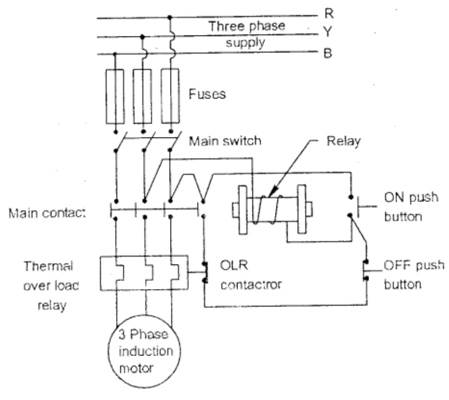


Fig 4.52 DOL Starter

By pressing the “ON” push button, the relay, in series between two phase gets energized. Hence the relay operator and closes the main contacts. Now the motor gets 3Q supply through OLR. Then motor starts to rotate. When the OFF push button is pressed, the relay is de energized and motor comes to stop. When overload occurs in the motor, terminal over load relay contactor melts, and hence the motor is disconnected from the supply.

Merits of D.O.L. Starter.

1. Simple in construction
2. Easy to install
3. Easy of maintain
4. In expressive

Demerits of D.O.L. Starter.

1. Used only small horse power motors
2. Starting current heavy.

4.20.2 **Rotor Resistance Starter :**

This type of starter is used to start slip ring induction motor. The slip ring induction motor is started by applying full line voltage to the stator windings. In the rotor circuit external resistance is included. The starting current is limited by introducing the variable resistance in each phase, of the rotor circuit. This arrangement increases the starting torque also.

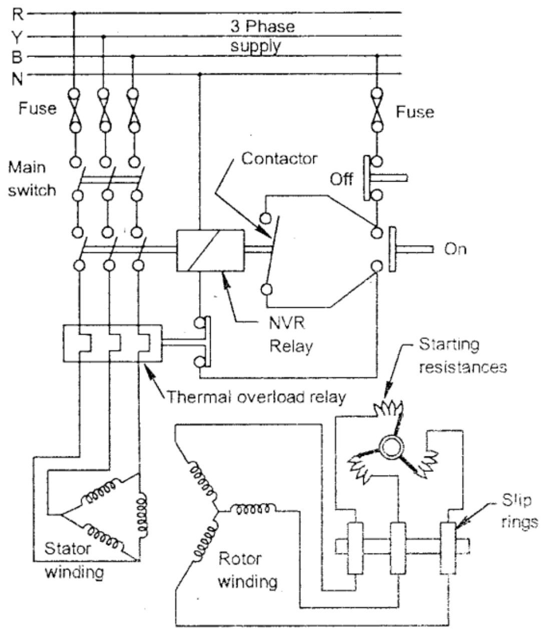


Fig 4.53 Rotor Resistance Starter

In the fig. 4.53, the fuses, relay, thermal overload relay are shown. In case if the motor is overloaded and unduly heated up. The thermal over load relay will operate and disconnected the motor from the supply.

In the rotor circuit, the controlling resistance provided may be of stud or contactor type. This may be hand operated (or) automatic, some means of inter locking arrangement are provided, of ensure proper operation of the line contactor and the rotor resistance. This meter locking provision helps to close the stator contactor , only if all the three phase resistance in the rotor circuit is included in the circuit.

Merits :

1. The Starting current is reduced
2. High starting torque is developed

4.20.3 AUTO-TRANSFORMER STARTER

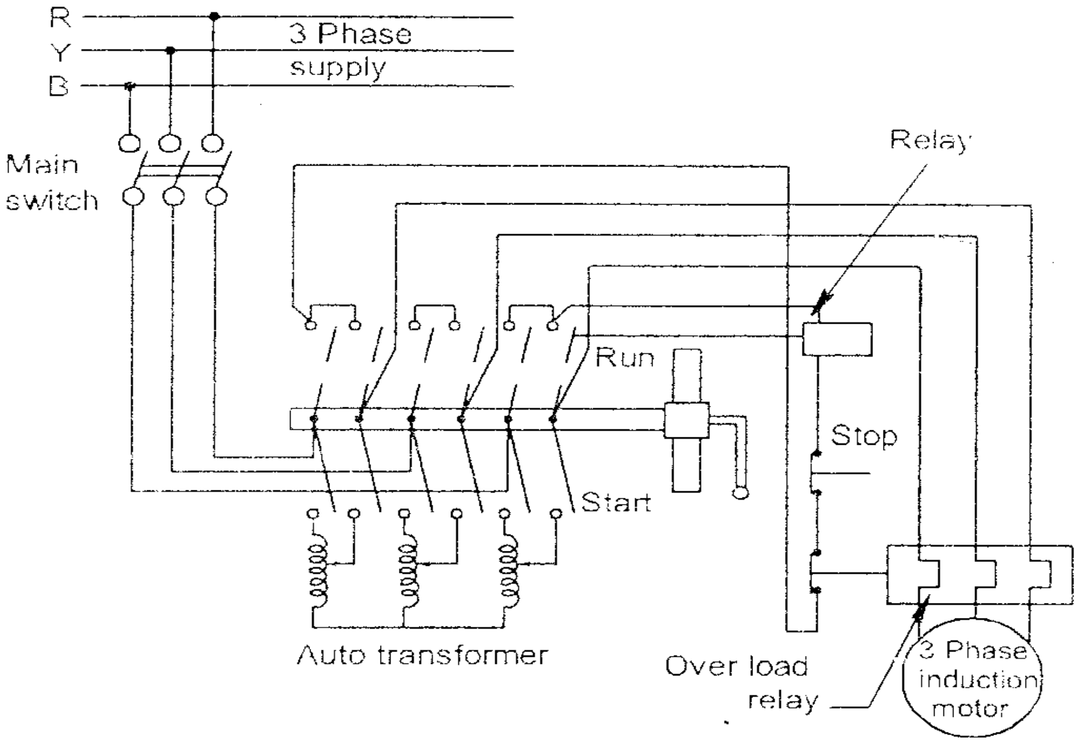


Fig 4.54 Auto Transformer Starter

This method of starting is suitable for cage motors. Auto transformer starter consists of 3 phase auto transformer with the provision of taps in order to give reduced voltage. The auto transformer are generally tapped at 50%, 60%, 80%, at about 80% of normal speed of the motor.

The manually operated type is having a multiple double throw switch as shown in fig4.54. During start, the switch is thrown to start position tapings of auto transformer. After the motor reaches about 80% of rated speed, the switch is changed over to "RUN" position and the motor is applied with FULL voltage from the mains the starter is provided with no volt release and over load release as protective devices.

No Volt Release :

If the supply fails (or) voltage drop in the line below a certain level, the iron piece inside the relay is demagnetized, and hence the relay contactor and supply contactor return to their original position, so the supply given to the motor is disconnected.

Over Bad Release :

Due to over load, the motor may get heated up due to this heat the thermal overload relay contactor melts, there by disconnecting the motor from the supply, this over load relay protects the motor from overload.

Merits :

1. Availability of the highest torque per ampere of supply cement.
2. Adjustment of starting voltage by selection of proper tap on the auto transformer
3. Stability for long starting period.

Demerits:

This type of starters is low power factor and higher cost.

4.20.4 **STAR DELTA STARTER :**

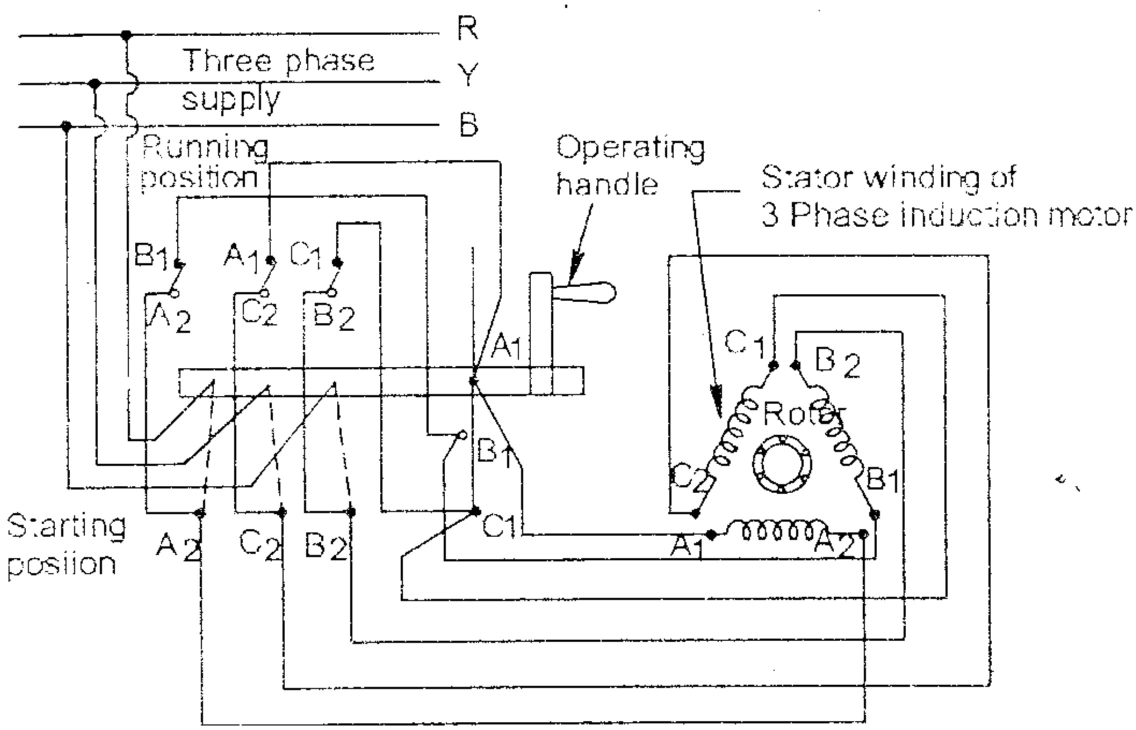


Fig 4.55 Star Delta Starter

The star delta starter connects the three stator windings in star across the supply voltage. After motor attains speed, the same through a change over switch, the connection diagram for star delta starter as show in fig.

Since at starting the stator winding are connected in star, the voltage across each phase winding is reduced 1//3 of line voltage (since in star Vp=L/3) . There fire the starting current is reduce to 1/3 times that of current taken with direct starting. The starting torque is also reduced to 1/3rd of starting torque obtained with direct switching.

No Volt Release :

If the supply falls on voltage drops in the line below a certain level the iron piece inside the relay is demagnetized and hence relay contactor and supply return to the original position. So the motor is disconnected from the supply.

Over Load Release

Due to overload, the motor may get heated up. Due to this heat, the Overload relay contactor melts thereby disconnecting the motor from the supply. Thus overload relay protect the motor from overload.

Merits :

1. This method of starting is simple, cheap and effective.
2. No power is wasted in auxiliary components.

Demerits :

This type of starting is unsuitable for line voltage exceeding 3000 volt.

4.21 **Slip :**

The difference between the synchronous speed Ns and the actual speed Nr of the rotor is known as slip. It is always expressed as the percentage of the synchronous speed.

$$\% \text{ slip} = \frac{Ns - Nr}{Ns} \times 100$$

Ns - Synchronous speed of motor

Ne - Actual speed of motor

(Ns-Nr) - Slip speed

$$\text{Slip (s)} = \frac{Ns - Ne}{Ns}$$

That fore

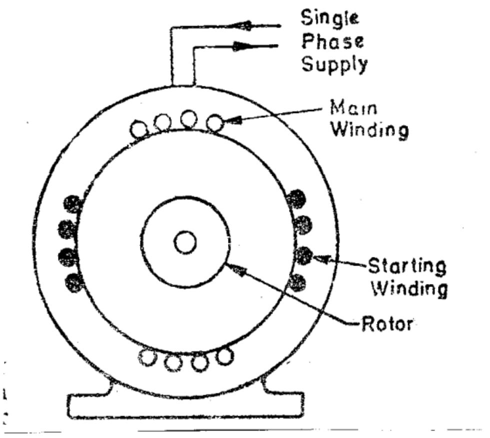
SNs = Ns-Nr

Nr = Ns – SNs

Nr = NS (1-s)

4.22 Single Phase Induction Motor

Single phase induction motor may be classified as under, depending on their construction and method of starting.



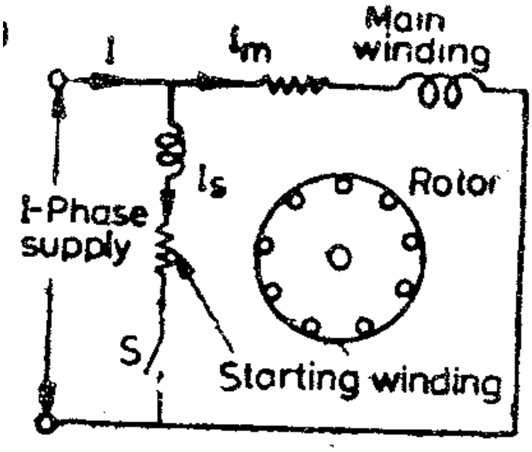


Fig 4.56 Fig 4.57

1. Induction motors – (split phase, coauthor and shaded pole etc.)
2. Repulsion motors – (sometimes called inductive series motor)
3. AC series motor
4. Un – excited synchronous motors – (Reluctance motor and Hysteresis motor)

4.22.1 **Single Phase Induction Motor** :-

A single phase motor consists of

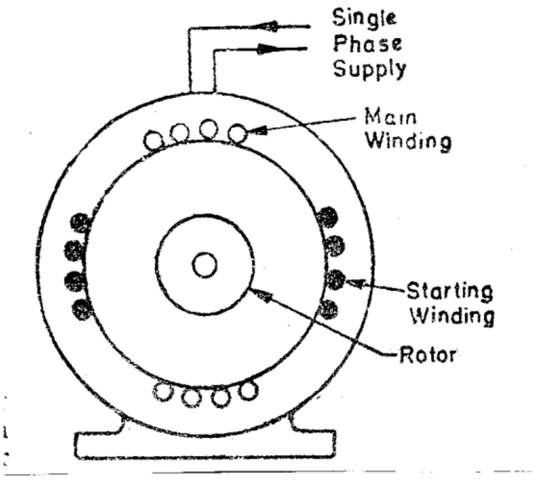
(i) A stator which carries single phase winding

(ii) A squirrel cage type of rotor

In the single phase motor is non self-starting and the single phase winding produces merely a pulsating (or) alternating flux. A synchronously rotating flux, can be produced only by either a 2-phase or 3-phase stator winding when energized from a 2-phase or 3-phase supply respectively.

An alternating flux acting on a stationary squirrel cage rotor cannot produce rotation (only a rotating flux can). However, if the rotor of a single phase motor is given as initial start by hand or otherwise in either direction then immediately a torque arises and the motor accelerates to its final speed provided, it is not heavily loaded.

Single phase Induction motor principle of operation.



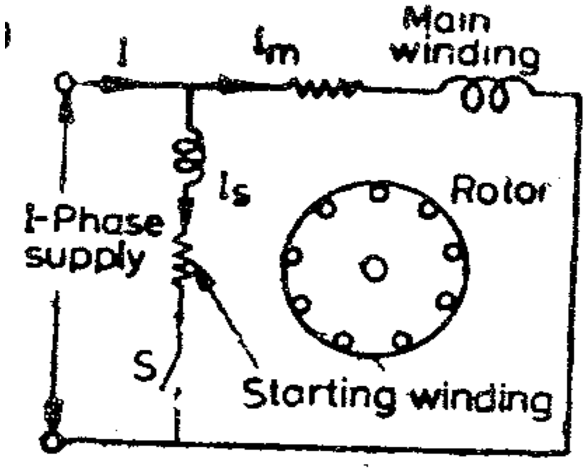


Fig 4.58. Fig 4.59

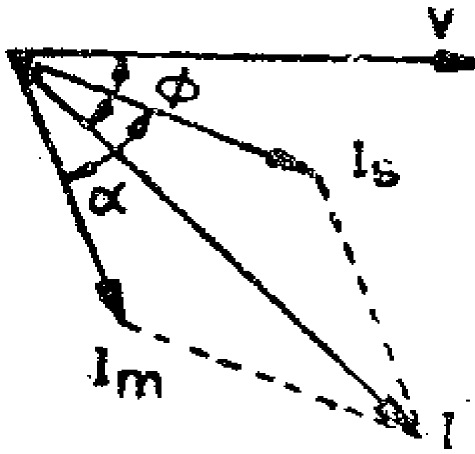


Fig.4.60

A single phase motor is temporarily converted into a 2-phase motor by providing an extra winding on the stator in addition to the main (or) running winding. The circuit connections are shown in fig 4.60. By making starting winding highly resistive and main winding highly reactive, the phase difference between the currents drawn by there can be made sufficiently large (the ideal value being 90`) . The motor behaves.

Applications

(1) Washing Machines
(2) Oil burners
(3) Blowers
(4) Wood working tools
(5) Bottle washers
(6) Graders and machine tools

It is generally available in the 50-25 walts range.

4.22.2 Capacitor Start Induction Motor

These motor have a higher starting torque, because in their case angle and between currents is and Im is large. The angle and is increased by connecting a capacitor in series with the starting winding as shown in fig.

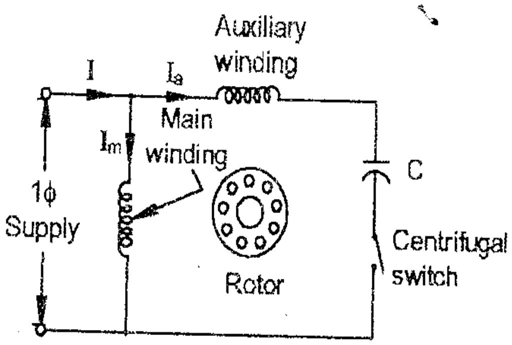
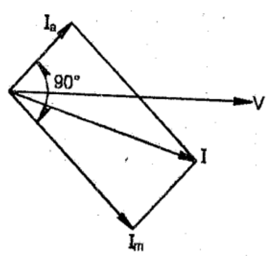


Fig.4.61



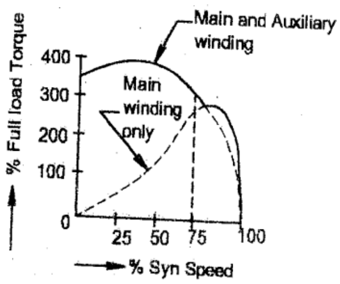


Fig.4.62

Usually the capacitor is mounted on top of the motor frame and is generally an electrolytic capacitor. As before the centrifugal switch cuts off both the starting winding and the capacitor when motor runs up to nearly 75 percent like a two phase motor for starting purpose fig.4.61 The two currents produce a revaluing flux and hence make the motor self starting. The starting torque $T = KI_s\ I_M \sin\alpha$ and where K is a constant governed by motor design parameters.

The centrifugal switch S is connected in series with the starting winding and is located outside the motor. Its faction is to automatically disconnect the starting winding from the supply when the motor reaches 70 to 80 percent of its full load.

Typical torque / speed characteristics for such a motor as shown in fig.4.63

The starting torque is 150 to 200 percent of the full load torque with a starting current of 6 to 8 times the full load current

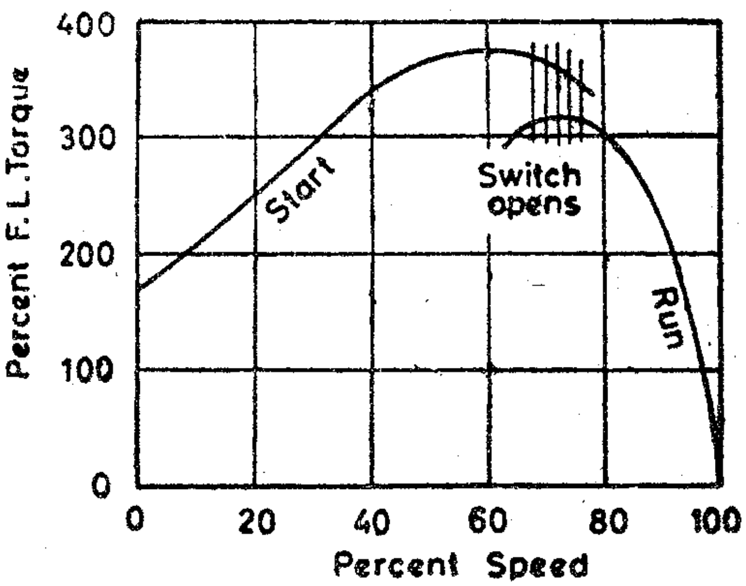


Fig 4.63 Torque Vs speed.

Of its full load speed. As seen from the fig the current is drawn by starting winding leads the voltage where as the current in Im in the main winding as before, lags V. In the way, value of and is increased to about 80* which increase the starting torque to twice that developed by a standard split phase induction. Such motors have starting torques as high as 450 percent of the full load value typical performance curve of such a motor as shown in fig. They are usually manufactured in the 100 – 500 W range.

The capacitors start motors are very popular for heavy duty general-purpose applications requiring high starting torque.

Application :

1. Compressors.
2. Jet Pumps
3. Form and Home-workshop tools
4. Swimming Tool pumps
5. Conveyors. Etc.,

4.23 STEPPER MOTOR :

Principle Operation of Stepper Motor :-

A stepper motor is basically a synchronous motor in stepper motor there is no brushes. This motor does not rotate continuously instead it rotates in form of pulses or in discrete steps. That's why it is called stepper motor.

There are different types of stepper motor.

1. Permanent Magnet
2. Variable Reluctance
3. Hybrid stepper motor

The first type is most important. The working and construction of the permanent magnet type stepper motor is as shown in fig.

The permanent magnet type motor has a stator that is of electromagnets and a rotor that is of permanent magnet. Therefore this motor is called permanent magnet type stepper motor.

Working :

When we gives supply to the stator, the winding of stator is energized and hence produces magnetic field Vs described above, the rotor is made up of permanent magnet, that why it tends to follow the revolving field, thus an stepper motor works.

The speed or torque of a permanent magnet type motor is changed by the number of poles used in stators. It we use a large number of poles in stator then the speed f motor will increase and if we use a less number of poles then the speed will decrease

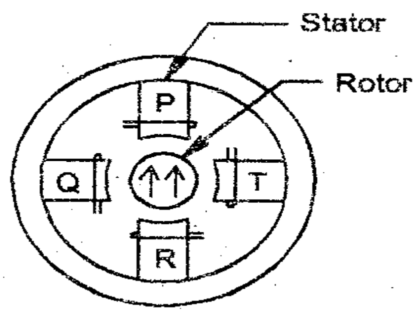


Fig.4.64

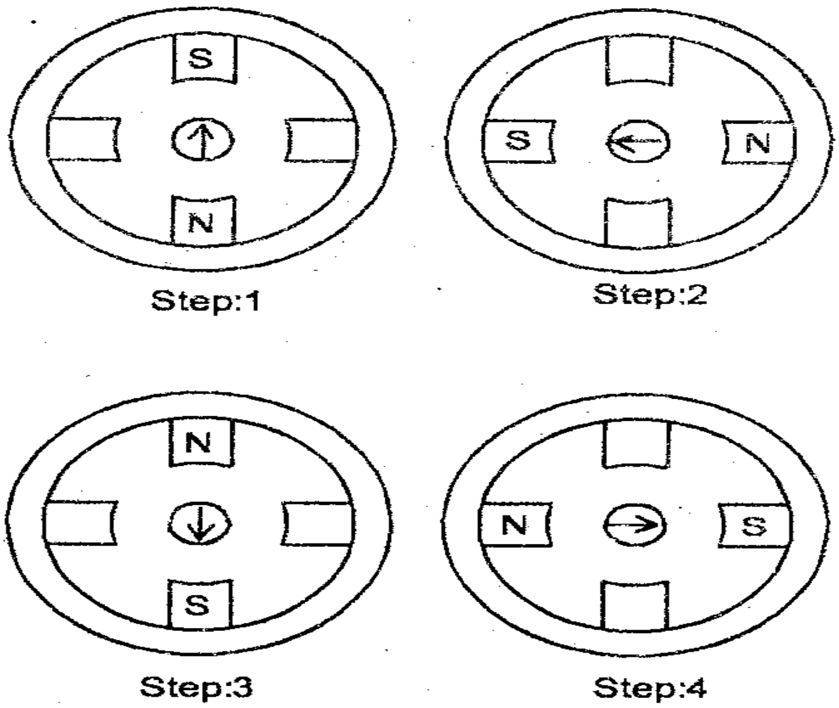


Fig.4.65

REVIEW QUESTIONS PART - A

1. Name the various part of a DC machine
2. What are the types of DC Generator ?
3. What is the Principle of DC Generator ?
4. What is the Function of Commutator ?
5. What is the Pole Pitch ?
6. Write the expression for emf equation of DC Generator
7. Define DC Generator
8. Define DC Motor
9. What is meant by back emf ?
10. What are Type of DC Motors
11. Define Alternator
12. Mention the type of Alternator
13. Define Slip
14. Define Induction Motor
15. Define stepper Motor

PART – B

1. What are the various types of DC Generator
2. What are the Application of a DC Generator
3. Mention the various application of DC Motor
4. What is the relation between speed and frequency
5. Mention the necessity of starter
6. Mention the various methods of starting of 3Q induction motor
7. Mention the application of capacitor start induction Motor

PART – C

1. Explain the working principle of DC Generator
2. Explain the working principle of DC Motor
3. Derive the emf equation of a transformer
4. Explain the working of 3 point starter
5. Explain the working of 4 point starter
6. Explain the working principle of 3Q alternator
7. Explain the principle of operation 3Q induction motor
8. Explain the principle of operation of single phase induction motor
9. Explain the principle of operation of capacitor start induction motor
10. Explain the principle of operation of stepper motor.

UNIT V

TRANSFORMERS

5.1 Transformer

Transformer is a static electrical machine which transforms electrical power from one circuit to another circuit, without changing in its frequency. The transformer works on the principle of electromagnetic induction.

Transformers are used to increase or decrease the alternating voltages in electric power applications.

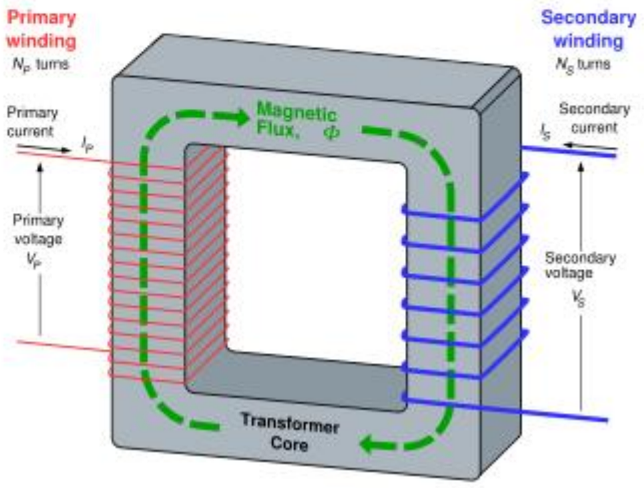


Fig 5.1

Transformers have two windings, being the primary winding and the secondary winding. The primary winding is the coil that draws power from the source. The secondary winding is the coil that delivers the energy at the transformed or changed voltage to the load. Usually, these two coils are subdivided into several coils in order to reduce the creation of flux.

In brief, transformer is a device that

1. transfers electric power from one circuit to another,
2. does so without change of frequency,
3. accomplishes this by electromagnetic induction and
4. where the electric circuits are linked by mutual induction.

5.2 Ideal Transformer

An **ideal transformer** is static electric machine which has
- no copper losses (no winding resistance)
- no iron loss in core
- no leakage flux

In other words, an ideal transformer gives output power exactly equal to the input power. The **efficiency of an idea transformer** is 100%.

In an ideal transformer, $V_1I_1 = V_2I_2$

Characteristics Of an Ideal Transformer :

- **Zero winding resistance**: It is assumed that, resistance of primary as well as secondary winding of an ideal transformer is zero. That is, both the coils are purely inductive in nature.
- **Infinite permeability of the core**: Higher the permeability, lesser the mmf required for flux establishment. That means, if permeability is high, less magnetizing current is required to magnetize the transformer core.
- **No leakage flux**: Leakage flux is a part of magnetic flux which does not get linked with secondary winding. In an ideal transformer, it is assumed that entire amount of flux get linked with secondary winding (that is, no leakage flux).
- **100% efficiency**: An ideal transformer does not have any losses like hysteresis loss, eddy current loss etc. So, the output power of an ideal transformer is exactly equal to the input power. Hence, 100% efficiency.

5.3 Principle of operation

The **working principle of transformer** is very simple. It depends upon Faraday's law of electromagnetic induction. When an input voltage is applied to the primary winding, alternating current starts to flow in the primary winding. The current flows, a changing magnetic field is set up in the transformer core. As this magnetic field cuts across the secondary winding, alternating voltage is produced in the secondary winding.

This emf is called 'mutually induced emf', and the frequency of mutually induced emf is same as that of supplied emf. If the secondary winding is closed circuit, then mutually induced current flows through it, and hence the electrical energy is transferred from one circuit (primary) to another circuit (secondary).

The ratio between the number of actual turns of wire in each coil is the key in determining the type of transformer and what the output voltage will be. The ratio between output voltage and input voltage is the same as the ratio of the number of turns between the two windings.

A transformers output voltage is greater than the input voltage if the secondary winding has more turns of wire than the primary winding. The output voltage is stepped up, and it is called as a **"step-up transformer**".

If the secondary winding has fewer turns than the primary winding, the output voltage is lower. This is a **"step-down transformer".**

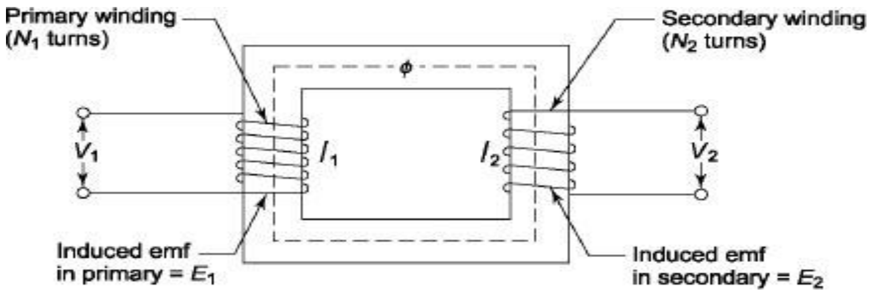


Fig 5.2

5.4 Construction Of Transformer

Basically a transformer consists of two inductive windings and a laminated steel core. The coils are insulated from each other as well as from the steel core. A transformer may also consist of a container for winding and core assembly (called as tank), suitable bushings to take out the terminals, oil conservator to provide oil in the transformer tank for cooling purposes etc. The above figure illustrates the construction of a transformer.

In all types of transformers, core is constructed by assembling laminated sheets of steel, with minimum air-gap between them. The steel used is having high silicon content and sometimes heat treated, to provide high permeability and low hysteresis loss. Laminated sheets of steel are used to reduce eddy current loss. To avoid high reluctance at joints, laminations are stacked by alternating the sides of joint. That is, if joints of first sheet assembly are at front face, the joints of following assemble are kept at back face.

5.4.1 Types Of Transformers

Transformers can be classified on different basis, like types of construction, types of cooling etc.

(A) On the basis of construction, transformers can be classified into two types as;

i) Core type transformer ii) Shell type transformer.

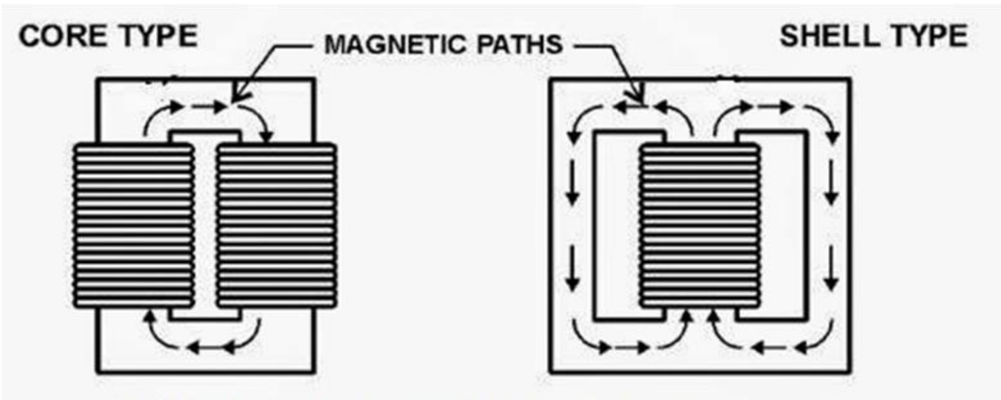


Fig 5.3

(I) Core Type Transformer

In core type transformer, windings are cylindrical former wound, mounted on the core limbs as shown in the figure above. The cylindrical coils have different layers and each layer is insulated from each other. Materials like paper, cloth or mica can be used for insulation.

(Ii) Shell Type Transformer

The coils are former wound and mounted in layers stacked with insulation between them. A shell type transformer may have simple rectangular form (as shown in above fig).

(B) On the basis of their purpose

1. Step up transformer: Voltage increases (with subsequent decrease in current) at secondary.
2. Step down transformer: Voltage decreases (with subsequent increase in current) at secondary.

(C) On the basis of type of supply

1. Single phase transformer
2. Three phase transformer

(D) On the basis of their use

1. Power transformer: Used in transmission network, high rating
2. Distribution transformer: Used in distribution network, comparatively lower rating than that of power transformers.
3. Instrument transformer: Used in relay and protection purpose in different instruments in industries
   - Current transformer (CT)
   - Potential transformer (PT)

5.5 Emf Equation of a Transformer

Consider a transformer having

N_1 = Number of turns in primary winding
N_2 = Number of turns in secondary winding
Φ_m = Maximum flux in the core (in Wb)
f = frequency of the AC supply (in Hz)

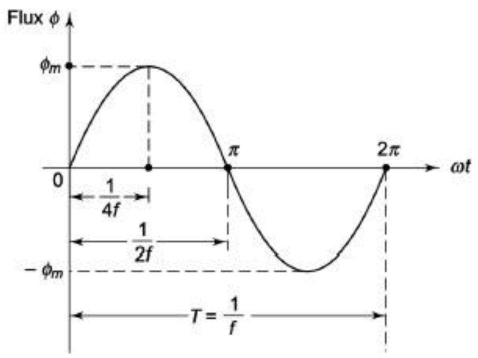


Fig 5.4

As, shown in the fig., the flux rises sinusoidally to its maximum value Φ_m from 0. It reaches to the maximum value in one quarter of the cycle i.e in T/4 sec (where, T is time period of the sin wave of the supply = 1/f).

Therefore,
average rate of change of flux = $\frac{\Phi m}{1/4f}$

$$= 4f\,\Phi_m \quad \text{(Wb/s).}$$

Now,
Induced emf per turn = rate of change of flux per turn

Therefore, average emf per turn = $4f\,\Phi_m$ (Volts).
As, the flux Φ varies sinusoidally,

Form factor of sine wave $= \frac{RMS\ value}{Average\ value} = 1.11$

RMS value of emf per turn = Form factor X average emf per turn.

Therefore, RMS value of emf per turn = $1.11 \times 4f\ \Phi_m$

$= 4.44f\ \Phi_m$.

RMS value of induced emf in whole primary winding (E_1)

= RMS value of emf per turn X Number of turns in primary winding

$E_1 = 4.44f\ N_1\ \Phi_m$ .... eq 1

Similarly, RMS induced emf in secondary winding (E_2) can be given as

$E_2 = 4.44f\ N_2\ \Phi_m$. .... eq 2

from the above equations 1 and 2,

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44f\ \Phi m$$

This is called the **emf equation of transformer**, which shows, emf / number of turns is same for both primary and secondary winding.

For an ideal transformer on no load,

$E_1 = V_1$ and $E_2 = V_2$.

where, V_1 = supply voltage of primary winding

V_2 = terminal voltage of secondary winding

Voltage Transformation Ratio (K)

Voltage transformation ratio $= \frac{E_2}{E_1} = \frac{4.44f\ N2\ \Phi m}{4.44f\ N1\ \Phi m} = \frac{N_2}{N_1} = \mathbf{K}$

Where, K = constant

This constant K is known as voltage transformation ratio**.**

- If $N_2 > N_1$, i.e. $K > 1$, then the transformer is called step-up transformer.
- If $N_2 < N_1$, i.e. $K < 1$, then the transformer is called step-down transformer.

Current transformation ratio (K)

In an ideal transformer,

$V_1 I_1 = V_2 I_2$

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

5.6 Losses in Transformer

As the electrical transformer is a static device, mechanical loss in transformer normally does not come into picture. We generally consider only electrical **losses in transformer**. Loss in any machine is broadly defined as difference between input power and output power. When input power is supplied to the primary of transformer, some portion of that power is used to compensate **core losses in transformer** i.e. **Hysteresis loss in transformer** and **Eddy current loss in transformer** core and some portion of the input power is lost as I^2R loss and dissipated as heat in the primary and secondary windings, because these windings have some internal resistance in them. The first one is called core loss or **iron loss in transformer** and the later is known as ohmic loss or **copper loss in transformer**. Another loss occurs in transformer, known as Stray Loss, due to Stray fluxes link with the mechanical structure and winding conductors.

Copper Loss in Transformer

These losses occur due to ohmic resistance of the transformer windings. If I_1 and I_2 are the primary and the secondary current. R_1 and R_2 are the resistance of primary and secondary winding then the copper losses occurring in the primary and secondary winding will be $I_1^2R_1$ and $I_2^2R_2$ respectively.

Therefore, the total copper losses will be

$$P_c = I_1^2R_1 + I_2^2R_2$$

These losses vary according to the load and known hence it is also known as variable losses. Copper losses vary as the square of the load current.

Core Losses in Transformer

Hysteresis loss and eddy current loss, both depend upon magnetic properties of the materials used to construct the core of transformer and its design. So these **losses in transformer** are fixed and do not depend upon the load current. So **core losses in transformer** which is alternatively known as **iron loss in transformer** can be considered as constant for all range of load.

Hysteresis loss in transformer is denoted as, $W_h = K_h f (B_m)^{1.6} \quad watts$

Eddy current loss in transformer is denoted as, $W_e = K_e f^2 K_f^2 B_m^2 \, watts$

Where, K_h = Hysteresis constant. K_e = Eddy current constant. K_f = form constant.

Core loss or Iron loss = Hysteresis loss + Eddy current loss

= constant losses

5.7 Transformer Efficiency

The **Efficiency** of the transformer is defined as the ratio of output power to the input power, the two being measured in the same unit. Its unit is either in Watts (W) or KW. Transformer efficiency is denoted by η.

$$\% \text{ Efficiency}(\eta) = \frac{\text{Output Power}}{\text{Input Power}} \times 100$$

$$= \frac{\text{Output Power}}{\text{Output power} + \text{Core loss} + \text{Copper loss}} \times 100$$

$$= \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_i + P_C} \times 100$$

Where output power = $V_2 I_2 \cos\theta_2$,

$\cos\theta_2$ = Power factor of load

Core loss P_i = Hysteresis loss + Eddy current loss

Copper loss $P_c = I_1^2 R_1 + I_2^2 R_2 = I_2^2 R_{es}$

5.7.1 Condition for Maximum efficiency

The transformer efficiency at a given load and power factor is given by the relation shown below

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_i + I_2^2 R_{es}}$$

Divide both numerator and denominator by I_2

$$\eta = \frac{V_2 \cos\theta_2}{V_2 \cos\theta_2 + P_i/I_2 + I_2 R_{es}} \text{ ------------ (1)}$$

The value of the terminal voltage V_2 is approximately constant. Thus, for a given power factor the Transformer efficiency depends upon the load current I_2. In the equation (1) shown above the numerator is constant and the transformer efficiency will be maximum if the denominator with respect to the variable I_2 is equated to zero.

$$\frac{d}{dI_2}\left(V_2\cos\theta_2 + \frac{P_i}{I_2} + I_2 R_{es} \right) = 0$$

$$0 \quad - \frac{P_i}{I_2^2} + I_2 R_{es} = 0$$

$$I_2^2 R_{es} = P_i$$

(i.e) Copper losses = Iron losses

Thus, the efficiency of a transformer will be maximum when the copper or variable losses are equal to iron or constant losses.

$$\eta_{max} = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + 2P_i} \quad \text{as } (P_c = P_i)$$

5.8 Voltage Regulation of Transformer

At a constant supply voltage the change in the secondary terminal voltage from no load to full load with respect to no load voltage is called as **Voltage Regulation** of a Transformer.

When the Transformer is loaded with a constant supply voltage, the terminal voltage changes depending upon the load and its power factor. The algebraic difference between the no-load and full load terminal voltage is measured in terms of voltage regulation. The change in secondary terminal voltage from no load to full load is $E_2 – V_2$. This change is divided by E_2(no load voltage) is known as 'regulation down'. If the change is divided by V_2 (full load voltage) then it is called 'regulation up'. Generally regulation is expressed in percentage.

$$\% \text{ Regulation down} = \frac{E_2 - V_2}{E_2} \times 100$$

$$\% \text{ Regulation up} = \frac{E_2 - V_2}{V_2} \times 100$$

5.9 O.C. and S.C. Tests on Single Phase Transformer

The efficiency and regulation of a transformer on any load condition and at any power factor condition can be predetermined by indirect loading method. In this method, the actual load is not used on transformer. But the equivalent circuit parameters of a transformer are determined by conducting two tests on a transformer which are,

1. Open circuit test (O.C Test)
2. Short circuit test (S.C.Test)

The parameters calculated from these test results are effective in determining the regulation and efficiency of a transformer at any load and power factor condition, without actually loading the transformer. The advantage of this method is that without much power loss the tests can be performed and results can be obtained. Let us discuss in detail how to perform these tests and how to use the results to calculate equivalent circuit parameters.

5.9.1 Open Circuit Test (O.C. Test)

The experimental circuit to conduct O.C test is shown in the Fig.1

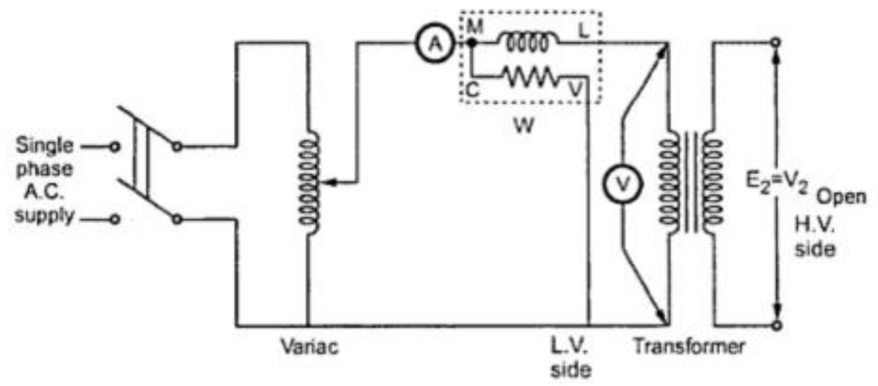


Fig 5.5 **Experimental circuit for O.C. test**

The transformer primary is connected to a.c. supply through ammeter, wattmeter and variac. The secondary of transformer is kept open. Usually low voltage side is used as primary and high voltage side as secondary to conduct O.C test.

The primary is excited by rated voltage, which is adjusted precisely with the help of a variac. The wattmeter measures input power. The ammeter measures input current. The voltmeter gives the value of rated primary voltage applied at rated frequency.

Sometimes a voltmeter may be connected across secondary to measure secondary voltage which is $V_2 = E_2$ when primary is supplied with rated voltage. As voltmeter resistance is very high, though voltmeter is connected, secondary is treated to be open circuit as voltmeter current is always negligibly small.

When the primary voltage is adjusted to its rated value with the help of variac, readings of ammeter and wattmeter are to be recorded.

The observation table is as follows

V_o volts	I_o amperes	W_o watts
Rated		

V_o = Rated voltage

W_o = Input power

I_o = Input current = no load current

As transformer secondary is open, it is on no load. So current drawn by the primary is no load current I_o. The two components of this no load current are,

$I_m = I_o \sin \Phi_o$

$I_c = I_o \cos \Phi_o$

where $\cos \Phi_o$ = No load power factor

And hence power input can be written as,

$W_o = V_o I_o \cos \Phi_o$

The phasor diagram is shown in the Fig. 5.6.

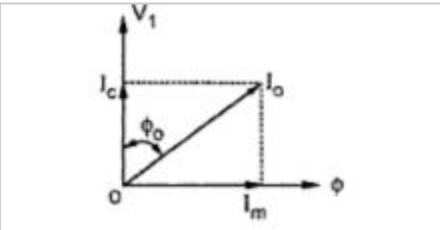


Fig 5.6 Phasor Diagram at No-Load

As secondary is open, $I_2 = 0$. Thus its reflected current on primary is also zero. So we have primary current $I_1 = I_o$. The transformer no load current is always very small, hardly 2 to 4 % of its full load value. As $I_2 = 0$, secondary copper losses are zero. And $I_1 = I_o$ is very low hence copper losses on primary are also very very low. Thus the total copper losses in O.C. test are negligibly small. As against this the input voltage is rated at rated frequency hence flux density in the core is at its maximum value. Hence iron losses are at rated voltage. As output power is zero and copper losses are very low, the total input power is used to supply iron losses. This power is measured by the wattmeter i.e. W_o. Hence the wattmeter in O.C. test gives iron losses which remain constant for all the loads.

$\therefore \quad W_o = P_i =$ Iron losses

Calculations : We know that,

$W_o = V_o I_o \cos \Phi$

$\cos \Phi_o = W_o / (V_o I_o)$ = no load power factor

Once $\cos \Phi_o$ is known we can obtain,

$I_c = I_o \cos \Phi_o$

and $\quad I_m = I_o \sin \Phi_o$

Once I_c and I_m are known we can determine exciting circuit parameters as,

$R_o = V_o / I_c \;\; \Omega$

and $\quad X_o = V_o / I_m \;\; \Omega$

5.9.2 Short Circuit Test (S.C. Test)

In this test, primary is connected to a.c. supply through variac, ammeter and voltmeter as shown in the Fig. 5.7.

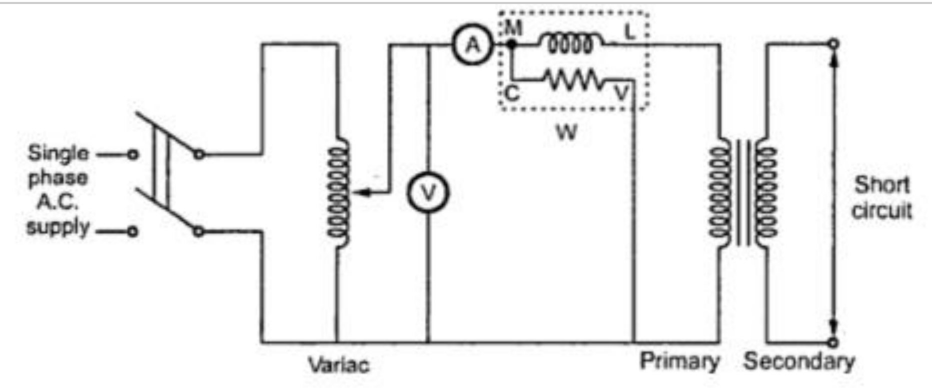


Fig 5.7 **Experimental circuit for S.C. test**

The secondary is short circuited with the help of thick copper wire or solid link. As high voltage side is always low current side, it is convenient to connect high voltage side to supply and shorting the low voltage side.

As secondary is shorted, its resistance is very very small and on rated voltage it may draw very large current. Such large current can cause overheating and burning of the transformer. To limit this short circuit current, primary is supplied with low voltage which is just enough to cause rated current to flow through primary which can be observed on an ammeter. The low voltage can be adjusted with the help of variac. Hence this test is also called low voltage test or reduced voltage test. The wattmeter reading as well as voltmeter, ammeter readings are recorded. The observation table is as follows,

V_{sc} volts	I_{sc} amperes	W_{sc} watts
	Rated	

Now the current flowing through the windings are rated current hence the total copper loss is full load copper loss. Now the voltage supplied is low which is a small fraction of the rated voltage. The iron losses are function of applied voltage. So the iron losses in reduced voltage test are very small. Hence the wattmeter reading is the power loss which is equal to full load copper losses as iron losses are very low.

$\therefore$ $W_{sc} = (P_{cu})$ F.L. = Full load copper loss

Calculations : From S.C. test readings we can write,

$W_{sc} = V_{sc} I_{sc} \cos \Phi_{sc}$

$\therefore$ $\cos \Phi_{sc} = V_{sc} I_{sc} / W_{sc}$ = short circuit power factor

$W_{sc} = I_{sc}^2 R_{1e}$ = copper loss

$\therefore$ $R_{1e} = W_{sc} / I_{sc}^2$

while $Z_{1e} = V_{sc} / I_{sc} = \sqrt{(R_{1e}^2 + X_{1e}^2)}$

$\therefore$ $X_{1e} = \sqrt{(Z_{1e}^2 - R_{1e}^2)}$

Thus we get the equivalent circuit parameters R_{1e}, X_{1e} and Z_{1e}. Knowing the transformation ratio K, the equivalent circuit parameters referred to secondary also can be obtained.

5.10 TRANSFORMER ON NO LOAD

For an ideal transformer, we have assumed that there are no core losses and copper losses. For practical transformers, these two losses cannot be neglected. At no-load condition, the primary current is not fully reactive and it supplies

(ii) iron loss in the core, that is, hysteresis loss and eddy current loss and

(iii) (ii) very small amount of copper loss in the primary. There is copper loss in the secondary because it is an open circuit.

The no-load current lags behind V_1 by an angle θ_0, which is less than 90° (around 80°–85°). The no-load input power is given by

$$W_0 = V_1 I_0 \cos\theta_0$$

where $\cos\theta_0$ is the no-load power factor. Figure shows the no-load phasor diagram of a practical transformer. The no-load primary current (I_0) has the following two components:

- One component of I_0, that is $I_w = I_0 \cos\theta_0$ is in phase with V_1. Since Iw supplies the iron loss and primary copper loss at no load, it is known as active or working or iron loss component.
- The other component of I_0 that is, $I_\mu = I_0 \sin\theta_0$ is in quadrature with V_1. It is known as magnetizing component. Its function is to sustain the alternating flux in the core and it is wattless.

$$I_0 = \sqrt{I_w^2 + I_\mu^2}$$

$$\theta_0 = \tan^{-1}\left[\frac{I_\mu}{I_w}\right]$$

and the following points are most important:

- The no-load primary current is 1–5 per cent of full-load current.
- Since the permeability of the core varies with the instantaneous value of exciting or magnetizing current, the waveform of exciting or magnetizing current is not truly sinusoidal.
- Since I_0 is very small, the no-load copper loss is negligible. Hence, no-load input is practically equal to the iron loss in the transformer.
- Since core loss is solely responsible for shifting the current vector I_0, the angle θ_0 is known as *hysteresis angle of advance*.

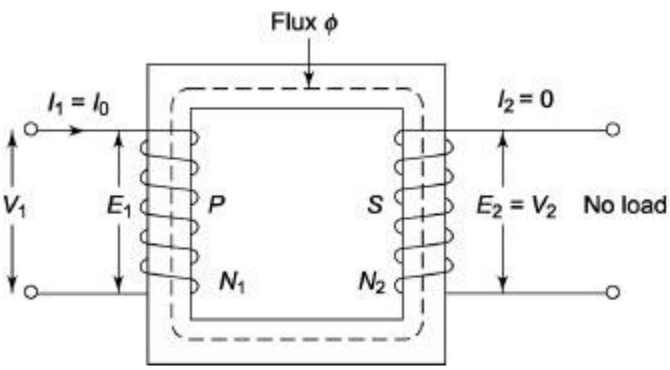


Fig 5.8 Equivalent Circuit of Transformer at No-load

5.11 TRANSFORMER ON LOAD

Figure5.9 (a) shows the transformer during no-load condition. The flux Φ is set up in the core. When the secondary is loaded shown in Figure (b), the secondary current will set up its own flux (Φ_2), which opposes Φ. The resultant flux becomes $\Phi - \Phi_2$. The value of e_1 will decrease because magnitude of Φ decreases. Hence, v_1 becomes greater than e_1 and the primary winding draws more current from the source. Let the additional current drawn by the primary be I_2'. This current I_2' will set up its own flux (Φ_2') in the same direction of Φ and it will oppose Φ_2 shown in Figure (c). The resultant flux will be Φ at any load condition if and only if

$\Phi_2' = \Phi_2$

i.e., $N_1 I_2' = N_2 I_2$

$$I_2' = \left[\frac{N_2}{N_1}\right] I_2 = \frac{I_2}{a}$$

i.e.,

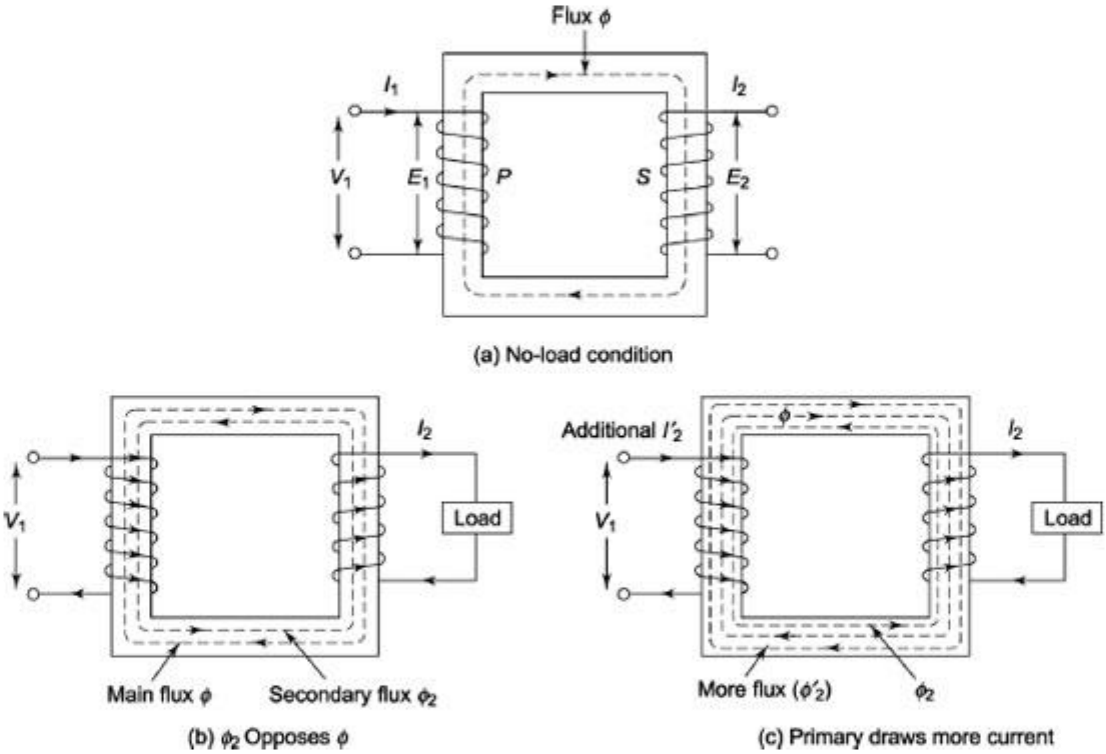


Fig 5.9 (a)No load (b) & (c)On-load Transformer

5.12 Auto Transformer

An **auto transformer** is an electrical transformer having only one winding. The winding has at least three terminals which is explained in the construction details below.
Some of the **advantages of auto-transformer** are that,

- they are smaller in size,
- cheap in cost,
- low leakage reactance,
- increased kVA rating,
- low exciting current etc.

An example of **application of auto transformer** is, using an US electrical equipment rated for 115 V supply (they use 115 V as standard) with higher Indian voltages. Another example could be in starting method of three phase induction motors.

Construction Of Auto Transformer

In **Auto Transformer**, one single winding is used as primary winding as well as secondary winding. But in two windings transformer two different windings are used for primary and secondary purpose. A diagram of auto transformer is shown below. The winding AB of total turns N_1 is considered as primary winding. This winding is tapped from point ′C′ and the portion BC is considered as secondary. Let's assume the number of turns in between points ′B′ and ′C′ is N_2.
If V_1 voltage is applied across the winding i.e. in between ′A′ and ′C′.

So voltage per turn in this winding is $\frac{V_1}{N_1}$

Hence, the voltage across the portion BC of the winding, will be,

$\frac{V_1}{N_1} \times N_2$ and from the figure above, the voltage is V_2

Hence, $\frac{V_1}{N_1} \times N_2 = V_2$

$\frac{V_2}{V_1} = \frac{N_2}{N_1}$ = Constant = k

As BC portion of the winding is considered as secondary, it can easily be understood that value of constant ′k′ is nothing but turns ratio or voltage ratio of that auto transformer.

When load is connected between secondary terminals i.e.between ′B′ and ′C′, load current I_2 starts flowing. The current in the secondary winding or common winding is the difference of I_2 & I_1.

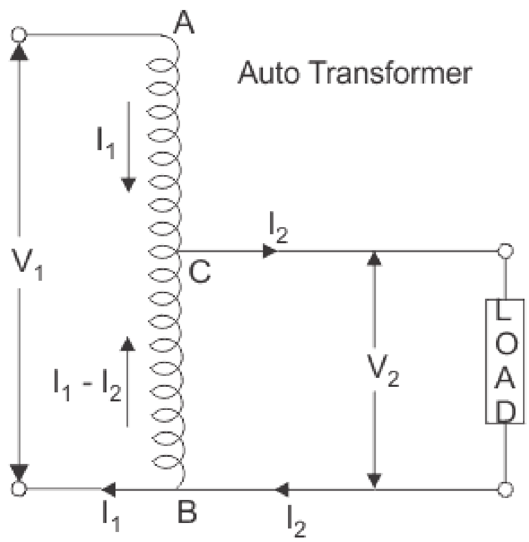


Fig 5.10

As winding is common in both circuits, most of the energy is transferred by means of electrical conduction and a small part is transferred through induction.

5.13 All day efficiency

All day efficiency is determined as, total KWh at the secondary to the total KWh at the primary of the transformer for a long specific time period preferably 24 hrs. i.e,

All day efficiency = $\frac{\text{Output in kWh}}{\text{input in kWh}}$

This is very much useful to judge the performance of a distribution transformer, whose primary is connected to the system forever, but secondary load varies tremendously throughout the day.

5.14 Applications of transformer

1. The transformer is used to either step up or step down the voltage.

2. It is used in power supply for electronic circuit.

3. In generating station, the transformer steps up the voltage for transmission.

4. It is used as an auto transformer starter for starting the induction motors.

5. It is used as an instrument transformer for increasing the range of meters.

Example 1 The voltage ratio of a single-phase, 50 Hz transformer is 5,000/500 V at no load. Calculate the number of turns in each winding if the maximum value of the flux in the core is 7.82 mWb.

Solution

Here

$E_1 = V_1 = 5{,}000$ V

$E_2 = V_2 = 500$ V

$\varphi_{max} = 7.82$ m Wb $= 7.82 \times 10^{-3}$ Wb, $f = 50$Hz

Let N_1 and N_2 be the number of turns of the primary and secondary windings, respectively.

Since

$E_1 = 4.44\, f\, \varphi_m N_1$

$$N_1 = \frac{E_1}{4.44\, f\phi_m}$$

i.e., $N_1 = \frac{5{,}000}{4.44 \times 50 \times 7.82 \times 10^{-3}} = 2880$ (since it is an integer number)

Again, $\frac{E_2}{E_1} = \frac{N_2}{N_1}$

$\therefore$ $N_2 = N_1 \times \frac{E_2}{E_1} = 2880 \times \frac{500}{5{,}000} \simeq 288$

Example 2 The no-load current of a 4,400/440 V, sinlge-phase, 50 Hz transformer is0.04. It consumes power 80 W at no load when supply is given to LV side and HV side is kept open. Calculate the following:

(i) Power factor of no-load current.
(ii) Iron loss component of current.
(iii) Magnetizing component of current.

Solution

W_0 W, I_0 =0.04 A, V_1 = 4,400 V

i. Since $W_0 = V_1 I_0 \cos\theta_0$

$$\cos\theta_0 = \frac{W_0}{V_1 I_0} = \frac{80}{4,400 \times 0.04} = 0.454$$

The no-load power factor is 0.454 (lagging).

ii. $I_W = I_0 \cos\theta_0 = 0.04 \times 0.454 = 0.0187$ A

iii. $$\sin\theta_0 = \sqrt{1 - \cos^2\theta_0} = 0.891$$

$\therefore I_\mu = I_0 \sin\theta_0 = 0.04 \times 0.891 = 0.0356$

UNIT- V

Review Questions

PART – A **(2 marks Questions)**

1. Define transformer ?
2. What is an ideal transformer?
3. What are the various losses in a transformer ?
4. What are the characteristics of an ideal transformer ?
5. What are the applications of transformer ?
6. Define turns ratio of a transformer.
7. Define efficienency of a transformer.
8. Define regulation of single phase transformer.
9. What is meant by step up transformer ?
10. What is meant by step down transformer ?
11. What is copper loss of a transformer ?
12. What is core loss of a transformer ?

PART –B **(3 Marks Questions)**

1. Derive the e.m.f .equation of a transformer .
2. Define current and voltage ratio of a transformer.
3. What is a Auto transformer ?
4. What are the advantage of auto transformer ?
5. How are transformers classified based on the construction ?
6. State lossess in transformer.

PART –C **(10 Marks Questions)**

1. Derive the condition for maximum efficiency of a transformer ?
2. Derive the e.m.f equation of a transformer.
3. Explain the open circuit test and short circuit test of a transformer ?
4. Explain the constructional details of a transformer ?
5. Explain efficiency and regulation of a transformer in detail.
6. Explain the working principle of a single phase transformer ? state its applications.
7. Explain transformer in no load condition in detail.
8. Explain transformer in load condition in detail.
